Impact of the Cn² description on WFAO performance

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Abstract. In the frame of the design of instruments for Extremely Large Telescopes, new techniques of Adaptive Optics have been developed. These techniques, generically called Wide Field Adaptive Optics (WFAO), are based on a tomographic reconstruction of the turbulent volume followed by a projection onto Deformable mirrors (DM) in order to ensure a good correction in a specified field of view (FoV). All these systems require a representation of the turbulent volume through the knowledge of the Cn² profile. It matters both for an accurate simulation of the input perturbations in the case of performance analysis and system design, but also for an efficient model description in the tomographic reconstruction process. We discuss and analyze the impact of the structure and the complexity of the real Cn² profile onto the WFAO performance. We demonstrate that a classical integrated parameter is not sufficient and that a more complex criterion is mandatory. Then, we focus on the impact of Cn² model error in the tomographic reconstruction process with respect to the input profile. We demonstrate that number and position of layers are two critical parameters. In conclusion, we show that it is critical to have access to high resolution Cn² profile to ensure a good performance evaluation of a WFAO system.

1. Introduction

New Adaptive Optics (AO) systems are under development in the frame of the instrumentation for the new generation of instruments for the Extremely Large Telescopes (ELT) and the next generation for the very large telescopes. Different concepts have been proposed during the last decade, such as Multi-Conjugate AO (MCAO) [1, 2], Laser Tomography AO (LTAO) [3], and Multi-Object AO (MOAO) [4]. They are designed to provide different kinds of correction, which depend on the goal of the scientific instrument used for the observation. These new techniques, which are called in the following Wide Field AO (WFAO) techniques, allow to enlarge the corrected Field of View (FoV) but also to enlarge the part of the sky that can be corrected by the AO system.

They are all based on a tomographic reconstruction of the turbulence thanks to several Wave Front Sensors (WFSs) that analyze the turbulence in different directions. The reconstruction of the turbulence is based on a two-step process: first an estimation of the turbulence volume and then a projection of the correction on the Deformable Mirrors (DMs). The optimal estimation

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of the turbulence in a static way is performed with a Minimum Mean Square Estimator (MMSE) [5, 6]. The goal is to minimize the residual error variance of the phase in the pupil:

$$\sigma^2_{res} = \sum_{i=0}^{nobj} \langle \| \Phi^{turb}(\beta_i) - \Phi^{cor}(\beta_i) \|^2 \rangle, \qquad \textbf{(1)}$$

β are the directions of interest where the correction has to be performed. All β directions correspond to the Science FoV (SFoV) as illustrated by Fig. 1. The goal is to find the optimal reconstructor that allows the reconstruction of the true turbulence Φ^{turb} thanks to the measurements Φ_α^{mes} performed by the WFSs in the different directions of analysis denoted as α. All the directions α represent the Guide Star FoV (GSFoV), shown in Fig. 1. In the following, the phase φ is represented on a convenient modal basis, the Zernike basis. In a linear case, the correction phase is equal to $\Phi^{cor} = P^{SFoV} \varphi^{est}$ where P^{SFoV} is the projector of the phase in the SFoV onto the DMs used for the correction. φ^{est} is the estimated phase equal to $\varphi^{est} = W_\alpha^{tomo} \Phi_\alpha^{mes}$ where W_α^{tomo} is the tomographic reconstructor and Φ_α^{mes} is given by the wavefront analysis: $\Phi_\alpha^{mes} = P_\alpha^L \Phi^{turb} + w$, where P_α^L is the projector of the phase in the altitude layer L in the WFS direction α and w is the WFS noise. The minimization of the criterion given by Eq. (1) is then obtained with the optimal Minimum Mean Square Error (MMSE) reconstructor given by:

$$W_\alpha^{tomo} = \sum\nolimits^{kol} (P_\alpha^L)^T \left(P_\alpha^L \sum\nolimits^{kol} (P_\alpha^L)^T + \Sigma_w\right)^-, \qquad \textbf{(2)}$$

where - stands for the pseudo-inverse of the matrix. $\sum^{kol}$ is the covariance matrix of the phase in the turbulent layers and is defined thanks to the characteristics of the turbulence. It represents the *a priori* knowledge on the turbulence. Σ_w is the covariance matrix of the measurement noise.

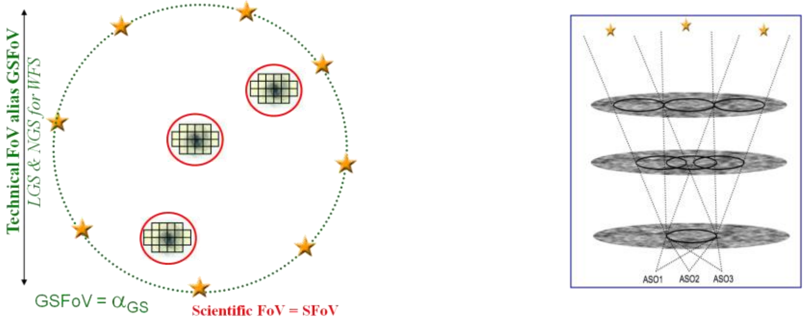

Fig. 1 Left: Illustration of the different FoV considered in WFAO (GSFoV and SFoV). Right: Tomography is performed with the use of several WFSs, which analyze the turbulence in different directions.

In the following, we focus our study on the parameters that have an impact on the performance of the tomographic reconstruction, and thus we do not compare the performance of the correction of the different WFAO systems. The tomographic reconstruction is the key of the performance in WFAO. Its goal is to obtain an estimation of the turbulent phase as close as possible to the true input turbulence in front of the instrument. The reconstructor given by Eq.

(2) allows a good reconstruction of the turbulence if the model used as a prior profile in the reconstruction process is close to the true input turbulence. The question is now: “how to have a good model of the turbulence, in the simulation of the true input turbulence itself and in the prior model used for the reconstruction?”

In section 2, we recall the definition of the main turbulent parameters and of the tomographic reconstruction error. Then in section 3, we present the impact of the true input turbulent profile used in simulation i.e. the impact of the error made on the simulation of the turbulent phenomenon. We show that the impact on the performance is important if we underestimate the strength and the distribution of the true input turbulence. In section 4, we study the impact of the errors on the prior turbulent model used in the reconstruction process. In this case, we give rise to the fact that an error on the repartition of the turbulent layers or on their strength has not the same impact on the performance.

2. Tomography related parameters and tomographic error term

The atmosphere which surrenders the telescope is a turbulent environment. Its description is based on the theory of the energy cascade proposed by Kolmogorov [7]. Atmospheric turbulence is usually considered as a succession of turbulent layers and can be described by some parameters:

- The altitude profile: the turbulence is considered as a succession of many discrete and independent layers, which thickness is assumed to be negligible. All these layers are distributed at different altitudes and form the altitude profile of the turbulence;
- The Cn^2 profile, which represents the repartition in altitude of the strength of the turbulent layers. This profile is directly linked to the altitude profile because it allocates to a given layer an energetic repartition;
- The Fried parameter r_0 [8] which illustrates the total strength of the turbulence:

$$r_0 = (0.42(2\pi/\lambda)^2 * 1/\cos\gamma \int_0^\infty Cn^2(h)dh)^{-3/5}. \tag{3}$$

This parameter is directly linked to the seeing $s = \lambda/r_0$.

In WFAO, we focus on the impact of the turbulence in a large FoV. That’s why we also consider the isoplanatic angle θ_0 [9], where

$$\theta_0 = 0.314\, r_0/\hbar \quad \text{and} \quad \hbar = (\int_0^\infty h^{5/3} Cn^2(h)dh / \int_0^\infty Cn^2(h)dh)^{3/5} \tag{4}$$

$\hbar$ is an equivalent altitude. θ_0 corresponds to the angular distance from the optical axis where the estimation error of the turbulence is lower than 1 rad² in variance. This parameter is linked to the fact that the turbulence spatially evolves.

Different techniques are used to obtain the true input Cn^2 profile: use of ground instruments such as SLODAR [10], or MASS [11, 12], measurements of the star scintillation with DIMM instruments [13], balloon measurements [14]. These techniques allow the measurement of the repartition of the true altitudes of the turbulence and their strength. The Cn^2 profile is a continuous function of altitude layers and can vary a lot spatially during the time.

The Cn^2 profile of the turbulence impacts the global system study at two levels, as illustrated by Fig. 2. It first impacts on the model chosen to simulate the true input turbulence. In fact, we can not use a continuous function of the Cn² because the number of parameters is too important. So in simulation, we define a true input Cn² profile which represents the true input turbulence, made of a given number N of discrete and independent layers. The problem is thus to find a reasonable sampling to model the perturbation introduced by the atmosphere. We also need to model the turbulence in the reconstruction process and to determine the prior turbulent profile used in the tomographic reconstructor, with N_L layers at different altitudes and a given prior Cn² profile. This prior turbulent profile can be obtained thanks to identification procedure used during the AO control or thanks to external measurements of the real turbulent profile. These two techniques allow an update of the model used in the reconstruction process in order to obtain a model as close as possible to the true Cn² profile.

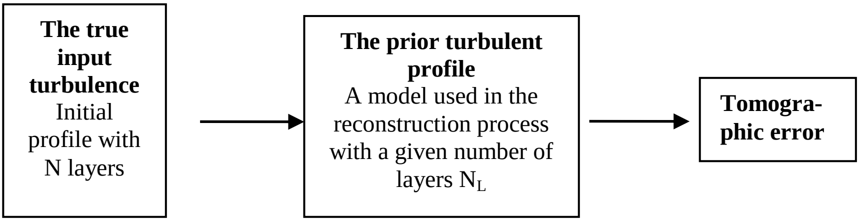

Fig. 2. The global system study for tomographic reconstruction.

In the following, we study the tomographic reconstruction process and its performance defined as the tomographic reconstruction error, $\sigma_{\text{tomo, recons}}^2$:

$$\sigma_{tomo,\ recons}{}^2 = \sum_{i=0}^{nobj} \| \Phi^{turb}(\beta_i) - \Phi^{recons}(\beta_i) \|^2, \tag{5}$$

where Φ^{turb} is the true input turbulence in the pupil and Φ^{recons} is the estimated turbulence reconstructed with the tomography based on a prior profile of the turbulence. The true input turbulence can be seen as the projection on a continuous turbulent profile (i.e. composed by a infinite number of layers) in a specific direction: $\Phi^{turb}(\beta_i) = P_1^{SFoV}\ \varphi^{turb}$ where P_1^{SFoV} is the projector of the phase on a given number N of layers. The prior profile of the turbulence corresponds to the model used in the tomographic reconstruction to estimate the turbulence: $\Phi^{recons}(\beta_i) = P_2^{SFoV}\ W_\alpha^{tomo} \Phi_\alpha^{mes}$, where P_2^{SFoV} is the projector of the prior profile on a given number N_L of turbulent layers.

In the following, we study two cases:

- When $N = N_L$, and $P_1^{\text{SFoV}} = P_2^{\text{SFoV}}$, the reconstruction process is “optimal” and the tomographic error evolution is only due to the input data and the Cn² profile considered to simulate the true input turbulence. We thus do not introduce a model error in the reconstruction. The impact of the choice of the model used to simulate the true input turbulence is studied in section 3;
- When $N > N_L$ and $P_1^{\text{SFoV}} \neq P_2^{\text{SFoV}}$, a model error is introduced in the reconstructor itself and the tomographic process is no longer optimal. We have thus to consider what is the impact of the mis-knowledge of the key parameters of the turbulent profile (Cn² profile, layers altitudes, etc). This is considered in Section 4.

3. Impact of the input turbulent profile

3.1. Impact of Cn² under-sampling in the input turbulent profile

In the following, we consider a WFAO system for a 42 meter telescope which works at 1.65 µm. To limit the impact of the fitting error, we use a DM which pitch is 0.5 m: it corresponds to a typical case with 84 x 84 actuators. To perform the wave front analysis in volume, the system uses n_{gs} WFSs, which are pointed the direction of natural GSs. The number of WFSs can vary but it is at least equal to 3. The sub-aperture size of each WFS is 0.5x0.5m². It defines the level of the aliasing error term, due to the aliasing of the high frequencies on the lower ones. The GS noise level is equal to 1 rad² for each case studied in order to be in good signal to noise conditions. We also take into account the delay of the AO loop. In this article, a simple type I integrator law is considered associated to 500 Hz sampling frequency WFS detector.

In the following, we assume that the true input Cn² profile is perfectly known. The prior Cn² profile is then equal to the true input Cn² profile, so that $P_1^{SFoV} = P_2^{SFoV}$.

We consider three types of Cn² profile (see Fig 3), each profile is simulated with the same r_0 and θ_0:

- Profile 1: the Cn² profile is made of N turbulent layers, N variable (between 2 and 100). The strength of the turbulence is equally distributed on each layer, so that the profile is considered as a constant profile;
- Profile 2: the Cn² profile corresponds to an Hufnagel profile [15];
- Profile 3: the Cn² profile is obtained from balloon measurements performed at Paranal.

The isoplanatic angle θ_0 is kept constant and equal to 1.7'. The seeing is equal to 0.8 arcseconds at zenith at 0.5 µm. The number of the GSs and their positions i.e. the size of the GSFoV can vary.

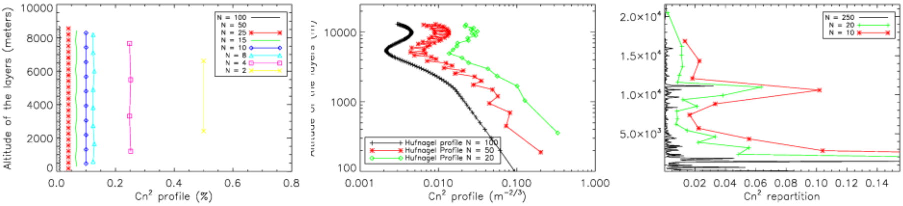

Fig 3. Shape of the different turbulent profiles used in the simulation. From left to right: constant Cn² profile, Hufnagel type profile and profile deduced from balloon measurements.

To calculate the impact of the under-sampling of the turbulence on the performance, we use a Fourier simulation code [16]. We introduce a true input turbulence with different number of layers and we assume that the Cn² profile is perfectly known in the reconstruction process ($N = N_L$). Results are presented in Fig 4. It is shown the tomographic error as a function of the number of layers in the true input turbulent profile. Using only a few numbers of layers to describe the true Cn² profile seems to be optimistic in terms of tomographic error. When the number of layers increases, the tomographic error increases as soon as it reaches a static stage. It is also important to note that the performance depends on the structure of the Cn² profile: we can see that even if the r_0 and θ_0 are the same for all types of profiles (for few layers in the profile same tomographic error), the shapes of the curves are different when the number of turbulent layers increases. The tomographic error is more important when the profile is obtained from balloon measurements than for a constant profile. Between them, there is 160 nm rms of additional tomographic error. As illustrated by Fig 3, Paranal and Hufnagel profiles look similar but we can see that there is 90 nm rms of additional tomographic error between these two profiles. This illustrates the importance of having accurate statistics of Cn² profile with high resolution data (in altitude) for WFAO system design and site selection.

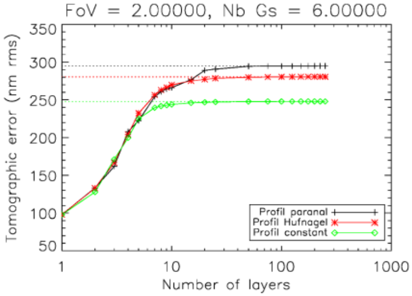

Fig 4. Impact of profile under-sampling on tomographic error for three different types of turbulent profiles (constant, Hufnagel and Paranal profiles).

3.2. Impact of system characteristics

Let us now focus on the impact of the system characteristics (number of GSs and GSFoV) on the performance of the tomographic reconstruction. We use a Hufnagel profile (profile type 2). As illustrated by Fig 5, we have a classic result: the tomographic reconstruction is better when we use lots of GSs in the FoV, and when these GSs are put on a small FoV. We also see that the beginning of the plateau depends on the size of the GSFoV: the larger the FoV is, the more you have to put turbulent layers in your input turbulent model in order to not under-estimate the tomographic error. In practice, at least 10 to 20 layers reconstruction layers are required to perform a efficient tomographic reconstruction for wide field AO systems.

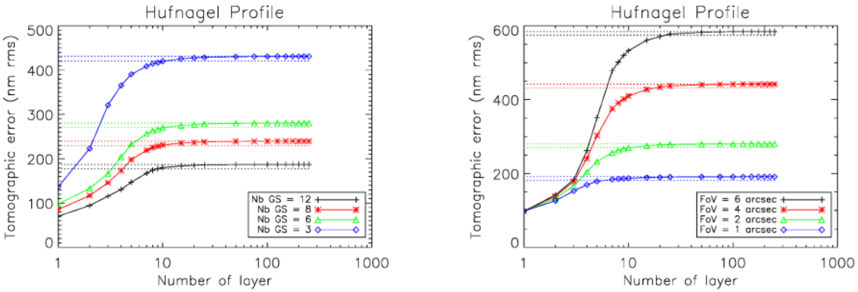

Fig 5. Impact of the system characteristics on the tomographic error for a Cn² constant profile. On the left, the number of GS varies when the GSFoV is equal to 2’, on the right the number of GS is equal to 6 and the GSFoV varies.

4. Model error in the tomographic reconstructor

In this section, we study the impact of a model error in the prior turbulent profile i.e. the case where $N > N_L$ and $P_1^{SFoV} \neq P_2^{SFoV}$. We study several *a priori* impacts: i) the error on the number of layers in the prior profile, ii) the error of the position of the turbulent layers and iii) the error of the Cn² repartition. For a sake of simplicity, we study the impact for the profile 1 type turbulence (constant profile) and we keep the same conditions: our WFAO system has 6 GSs put on a 1' or 2' diameter circle. First we study the impact of an error of the number of layers chosen to represent the prior turbulent profile. As illustrated by Fig 6, the tomographic error decreases with the increase of the number of layers in the prior turbulent profile. If you put few turbulent layers in the model of the reconstructor, the tomographic error is over-estimated. In fact, these results show that at least more than 20 layers are needed in the prior turbulent model to have a good estimation of the performance.

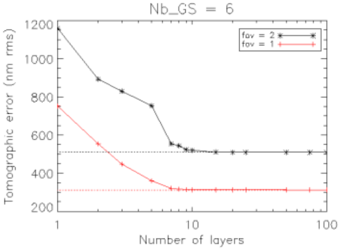

Fig 6. Impact of the mis-knowledge of the number of layers in the input turbulent profile ($N \neq N_L$).

We also study the impact of an error of the altitude of the layers and of the Cn² repartition. We only look at the impact in a case where the true input turbulent profile and the prior turbulent model have the same number of layers ($N=N_L$). For both model errors, we add a growing error directly on the model used to simulate the true input turbulent profile. Results are presented on Fig 7. For both cases, the impact of a model error is lower when we put lots of turbulent layers in the profile. But we can see that the impact on the tomographic error is different: when we make an error on the altitude, the impact is more important than if we make an error on the Cn² repartition: 160 nm rms for the worst altitude error and only 35 nm rms for the worst Cn² error. So increasing the number of layers in the model will relax the requirements on the knowledge of their absolute positions.

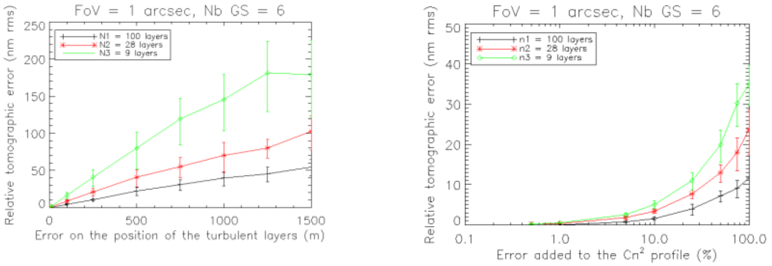

Fig 7. Impact of model error on the tomographic error. Left: error on the altitude of the layers, right: error on the Cn² repartition.

5. Conclusion

In this paper, we have studied the impact on the Cn² profile description on system design and performance assessment for WFAO as well as input for tomographic reconstructor. We emphasize the fact that the tomographic error is sensitive to the input Cn² profile. The larger the FoV is, the more you need turbulent layers in your input profile to avoid optimistic results in term of tomographic residual errors. We have also studied the impact on the performance of a mis-knowledge of the Cn² profile. Conclusions are quite the same as the previous study: it is important to have a good model of the Cn² profile as a prior information because it relaxes the requirements on the knowledge of their absolute positions. We show that an error on the altitude of the layers is more important than an error of the Cn² repartition. These two studies call for both high resolution Cn² profiles, better than a few hundred meters of resolution in altitude, as input for WFAO studies. These studies have to be completed. In particular, we need to confirm these results with an analytical study where a "WFAO criterion" could be propose to relate Cn² structure and repartition to the performance of the system.

6. References

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