# OPTIMAL CONTROL OF TIP-TILT MODES
# on-sky adaptive optics demonstration

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**Abstract.** An $\mathcal{H}_2$-optimal control approach for Adaptive Optics has been validated in an on-sky experiment on a solar telescope. A substantial performance improvement over the integrator control approach is demonstrated for control of the tip-tilt modes. The experimental results correspond reasonably well with simulations based on measured data.

## 1 Introduction

In recent years various researchers have proposed to apply optimal control methods to the Adaptive Optics (AO) system. Optimal controllers are expected to achieve a better performance than the commonly used type of AO controller, which is based on an integrator. This improved performance is achieved by taking into account the temporal and spatial correlation properties of the wavefront phase disturbances. Moreover, an optimal control can properly deal with the dynamical behaviour of the AO system, in particular the dynamics of the deformable mirror and the wavefront sensor. An AO integrator controller normally does not account for the specific wavefront phase correlation properties nor the mirror dynamics. The proposed control approaches as described in [1–5] share the objective to minimize a quadratic criterion function of the wavefront phase error, such that the overall Strehl ratio will increase. This objective renders so-called optimal or LQG control solutions, which are in some cases also referred to as 'predictive controllers'. On a detailed level the proposed methods for AO optimal control differ in several aspects, such as the initial assumptions on the disturbance characteristics, the behaviour of the deformable mirror (DM) and the 'domain of control', zonal, modal or Fourier for instance. For the experiments described in this paper we have followed the approach by Hinnen [5]. The main philosophy behind this approach is that it is fully *data driven*. This means that no initial assumptions are made on properties such as the spatial correlation, modal decoupling, possibly frozen turbulence flow or the (low) order of disturbance models. The design of the optimal controller is completely determined by the statistics of the measured wavefront sensor signals.

The standard system set-up of $\mathcal{H}_2$-optimal control design is depicted in Fig. 1. Here $\mathcal{P}$ represents the

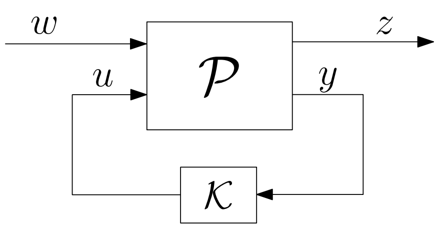


**Fig. 1.** General system set-up of an $\mathcal{H}_2$-optimal control problem.

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generalized plant and $\mathcal{K}$ is the controller. The output $z$ is the error signal, $y$ is the measured output signal, $u$ is the control input signal and the signal $w$ contains all external inputs to the system, including disturbances and sensor noise. The control law is:

$$u(k) = -\mathcal{K}y(k) \tag{1}$$

in which $k$ is the time instant.
Note that in this generalized plant case the measured output signal $y$ differs from the error signal $z$ and that $y$ may not contain full information of $z$. For the specific case of AO control, this system property is embodied by having the objective to minimize the wavefront phase error which, however, cannot be measured directly by the wavefront sensor. In [5] and [6] the specifics of the AO problem in the frame of $\mathcal{H}_2$-optimal control are worked out in further detail.
The $\mathcal{H}_2$ control problem is to design a controller $\mathcal{K}$, that stabilizes the overall plant $\mathcal{P}$ and minimizes the root-mean-squared value of $z$. This can be shown to be equivalent to the minimisation of the $\mathcal{H}_2$-norm of the transfer matrix from $w$ to $z$. The criterion function for the AO case in particular contains the variance of the wavefront phase error $\varphi$ plus an additional weighting term on the control effort:

$$\mathcal{J} = \mathrm{E}\left\{\varphi^T(k)\varphi(k) + u^T(k)Qu(k)\right\} \tag{2}$$

in which the matrix $Q$ can be tuned to improve the robustness of the control solution.
Finding the controller $\mathcal{K}$ that stabilizes the plant $\mathcal{P}$ and minimizes the criterion $\mathcal{J}$ forms a standard optimal control design problem, which generally involves solving two Riccati equations; one for optimal state estimation and one for state feedback. For the system given above the reader is referred to the optimal control theory described in for instance [7] and [8].

**Fig. 2.** McMath-Pierce solar telescope; courtesy of NOAO/AURA/NSF.

## 2 Experimental set-up

In previous years favourable results have been achieved with the $\mathcal{H}_2$-optimal control approach both in simulations using measured wavefront sensor (WFS) data from telescopes [9] and in real-time experiments on an Adaptive Optics laboratory set-up [10]. These experimental steps are indispensable in the verification process of optimal AO control. However, both steps lack an important practical factor; being the real-time aspect (simulations with telescope WFS data) or true turbulence data (laboratory set-up). Therefore, the next step in verifying the performance of optimal AO control is to implement it on a telescope's AO system and run observation experiments with it. The McMath-Pierce solar telescope (Fig. 2) on Kitt Peak, Arizona is a very suitable location for the on-sky AO control experiments.

The telescope has an aperture of 1.5m and is still one of the largest solar telescopes in the world. The AO system at McMath-Pierce is designed to give diffraction-limited performance under medium seeing conditions for wavelengths larger than 2.3$\mu$. Its main components are a 37-channel electrostatic membrane mirror, a piezo-based tip-tilt mirror and an orthogonal lenslet array with a 256x256 pixel CCD camera as wavefront sensor. The AO control system is made up of an industrial Pentium PC with Linux as operating system. Further details of the AO system can be found in [11].

Originally, the verification plan aimed at controlling the first 20-30 modes of the AO system, using both the tip-tilt mirror and the deformable mirror. During the experiments, however, it appeared that the deformable mirror did not have sufficient stroke to fully cope with the strong wavefront aberrations as measured by the AO wavefront sensor on the particular days of the experiments. The graph below shows a histogram of normalised, absolute DM actuator signals that would be required for optimal control, in which the maximum allowed amplitude equals unity.

The specific measurements for this result were taken on November 12, 2010. It should be stressed

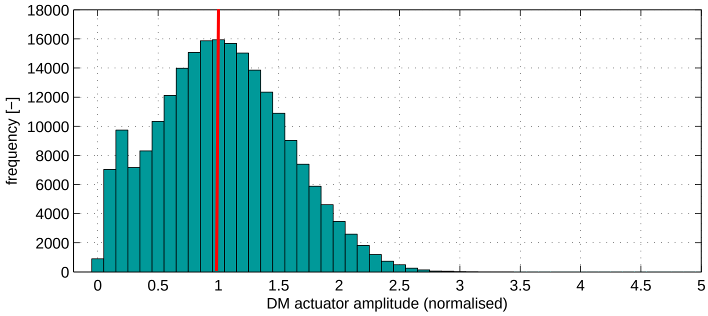


**Fig. 3.** Histogram of required DM actuator amplitudes for optimal control; the red line represents the maximum allowed amplitude.

that it represents a particular situation and that Fig. 3 does not have any generic validity. Nevertheless, based on Fig. 3 it was decided to focus on optimal control of the lower aberration modes tip and tilt only.

The optimal control method is a model-based approach and so it requires dynamic models of $\mathcal{P}$, in particular of a) the atmospheric turbulence-induced wavefront phase disturbance and b) the mirror-wavefront sensor system (the 'plant' in control terms). The essential philosophy of our control approach is that the dynamic models of both wavefront disturbance and the plant are data-driven. For the wavefront disturbance model this means that first a recording is made of the wavefront sensor data and consecutively a parametric model is estimated that fits best to the recorded data. The subspace identification procedure used for estimation of the dynamic models is described in [12]. As an illustration of the identification results in Fig. 4 the power spectral density (PSD) of the tip and tilt modes are shown. Here the disturbance model is a 4-th order state-space model. The PSD's of the models show a good match with the PSD's as calculated directly from the measured data. The proportion of variance accounted for (VAF) equals 92% in this case. Increasing the order of the disturbance model does not improve its accuracy. It should be noted that although the disturbance models may accurately represent the wavefront phase dynamics at the time of the measurement, due to the time-variant nature of the turbulence the models may become outdated. An optimal controller will therefore loose (part of) its performance as it is based on 'old' data. An adaptive controller may circumvent this and maintain the performance over longer observation intervals [6].

The model for the AO plant - i.e. the series connection of the tip-tilt mirror, the optical path, the wavefront sensor and the CCD data post-processing - has been identified by exciting the mirror with a broadband stochastic noise signal. The data-set consisting of both the input and output data (a synchronous recording of the excitation noise and the calculated tip-tilt amplitudes) is then processed by the subspace identification routine [12] to produce a dynamical model of the AO system. In Fig. 5 the frequency response is shown for the 2x2 AO tip-tilt system under closed-loop integrator control (4-th

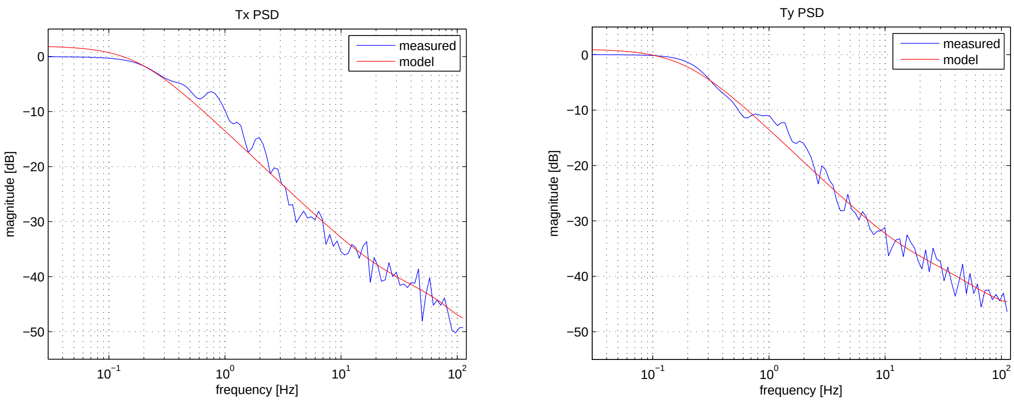


**Fig. 4.** Power spectral density of tip and tilt modes.

order state-space model). Clearly, for this system the diagonal components are dominant.

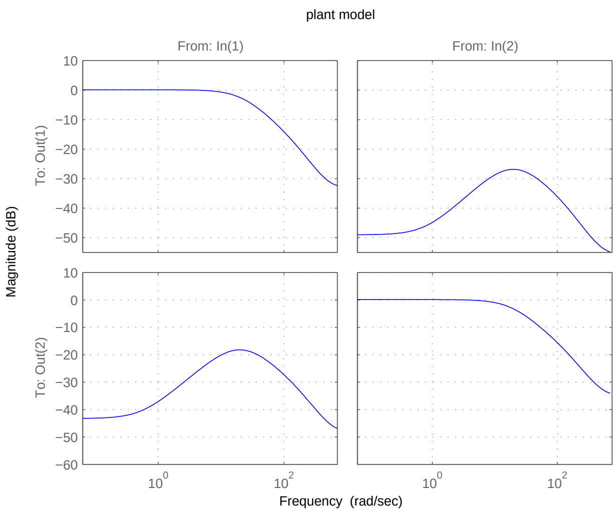


**Fig. 5.** Magnitude of the frequency response of the closed-loop 2x2 tip-tilt system.

## 3 Results

Based on the dynamic models for the tip-tilt disturbance and the AO system, the expected performance of the optimal controller can be simulated and also be compared to the performance of the common integrator controller. This result is shown in Fig. 6. Clearly, the optimal controller achieves a higher disturbance rejection in the low frequency band and also has a higher control bandwidth. In terms of the $\mathcal{H}_2$-norm the integrator controlled system has a norm of 0.24 whereas the optimal controlled system arrives at a norm of 0.11.

In a true observation experiment the optimal controller has been implemented on the AO bench of the McMath-Pierce telescope. As a reference in the wavefront sensing a sunspot has been used.

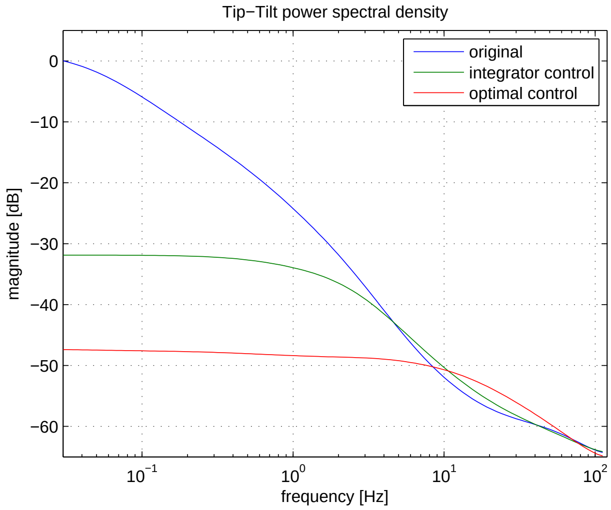


**Fig. 6.** Comparison of simulated control performances in terms of total tip and tilt PSD.

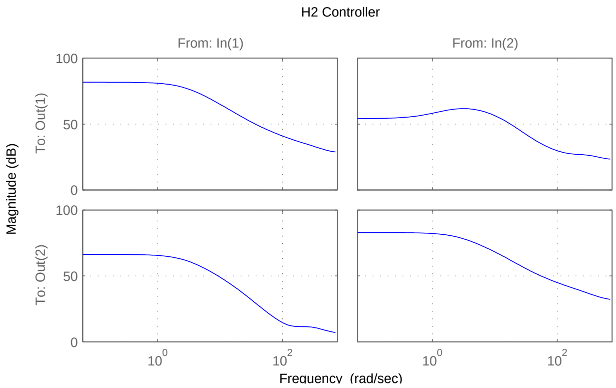


**Fig. 7.** Magnitude of the 2x2 controller frequency response.

The optimal controller has been designed to minimize a criterion function of the form of eq. (2), accustomed for the tip-tilt disturbance:

$$\mathcal{J}_2 = \mathrm{E}\left\{t_x^2(k) + t_y^2(k) + \rho u^T(k)u(k)\right\} \tag{3}$$

in which $t_x(k)$ and $t_y(k)$ are the amplitudes of the tip and tilt modes at time instant $k$. The weighting factor $\rho$ was set equal to 0.01. The properties of the resultant optimal controller are given in Fig. 7. The diagonal components dominate the control action.
Various recordings have been made of the wavefront sensor data (blocks of 24s at 250 Hz frame rate) with the tip-tilt mirror driven by the optimal controller, the integrator controller and without control. The performance results are shown in Fig. 8 in which for each case the PSD's represent the results averaged over 4 different experiments, carried out in a time frame of 2 hours. The specific experiments took place on November 14, 2010.

Also in the on-sky experiments the optimal controller shows a significantly better disturbance rejection in the lower frequency band together with a higher control bandwidth. This demonstrates the added value of a model-based optimized controller over a straightforward integrator. Still, in absolute sense the integrator and the optimal controller do not perform as well as in the simulation case (Fig. 6). In terms of tip-tilt amplitudes (RMS) the residual levels for the integrator and the optimal controller

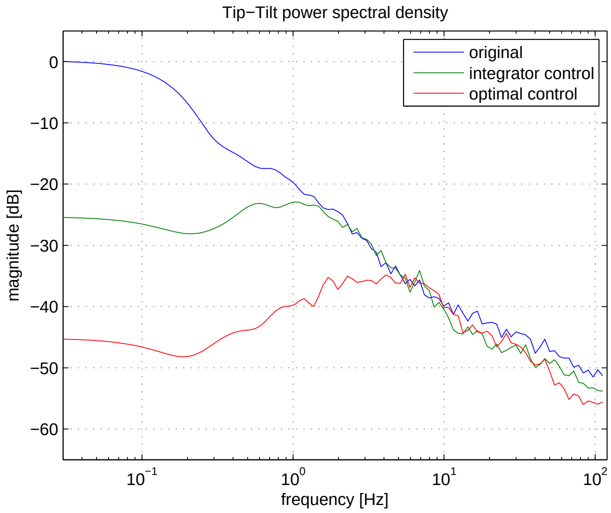


**Fig. 8.** Comparison of on-sky control performances (averaged over several tests) in terms of total tip and tilt PSD.

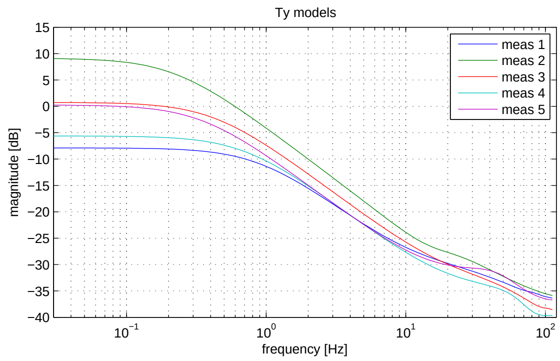


**Fig. 9.** Frequency responses of disturbance models for $T_y$.

are 0.46 and 0.30 (RMS) respectively, scaled to an original level of 1. Especially in the mid-frequency band the rejection of the tip-tilt disturbance is less than what is ideally achievable. Possible reasons for this are errors in either the plant model or the disturbance model. The time-variant character of the tip-tilt disturbance in particular could be a cause. Whereas the controller is tuned to the specific disturbance properties as measured a priori, it looses part of its performance once the disturbance characteristics vary with time. An impression of the variations in the tip-tilt disturbance is given in Fig. 9. It shows 5 disturbance models estimated from open-loop tip-tilt recordings taken in a time frame of 1 hour. Although, the general spectral behaviour remains unchanged, the magnitude curves show variations with time, especially with respect to the low pass cut-off frequency.

## 4 Conclusions

Optimal control for AO tip-tilt modes has been demonstrated on a solar telescope with on-sky turbulence. It obtains a substantial disturbance rejection and clearly outperforms the integrator control. The achieved performance agrees quite well with simulations. Similar performance results can be expected when optimal control is applied on higher order on-sky modes. Due to the time-varying turbulence properties the optimal controller looses part of its performance over longer observations. An adaptive control implementation could be beneficial to maintain the favourable rejection level.

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