Tomographic phase diversity for non-common path aberrations retrieval on wide field AO systems

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Abstract. Phase diversity is a commonly used technique to retrieve the wavefront at the focal plane. The usual algorithm involves two or more images of the same target with known phase changes like defocus. It has been shown to be very efficient at measuring on-axis the non-common path aberrations of classical AO systems. In this paper, we present an evolution of this algorithm towards tomographic measurements. This novel technique is dedicated to wide-field AO systems, allowing phase retrieval on multiple layers, conjugated at various altitudes. While the general grounds are very similar to classical phase diversity, the tomographic algorithm involves two or more images with known phase changes of several targets dispatched over the entire field of view. We additionally propose two versions for this algorithm: an image-based and a Fourier-based both leading to comparable results. We finally present the results obtained on simulated data as well as on real data obtained on GeMS: the Gemini MCAO system on which this algorithm has been used to estimate and compensate for non common path aberrations.

1 Introduction

Phase diversity is a focal plane wavefront sensing technique, introduced about twenty years ago. Its principle is based on the recording of several images with known phase variations between each other. One common application for this technique is the non common path aberrations (NCPA) retrieval for adaptive optics (AO) systems. It has been successfully used to do so on various AO systems (NAOS-CONICA, Keck OSIRIS, etc..) and substantial work has been done during the past decade to refine this technique. A usual application of phase diversity is the retrieval of both the phase and the observed object when it is not known [1]. In the following we concentrate on the retrieval of the phase only, the object being considered as point-like or with a known shape (e.g. Gaussian).

Classical phase diversity (i.e. using one reference source) is well suited to the case of non-varying aberrations across a given field of view (FoV). However, in the case of wide FoV AO systems, aberrations are considered to be varying across the FoV and the system is usually designed around several pupil plane wavefront sensors using the light from various reference sources in order to do tomographic wavefront sensing and reconstruction. A similar approach is thus required to compensate simultaneously over the whole FoV for NCPA on such systems. Kolb[2] has proposed an approach to correct for the Non-Common Path Aberrations in an MCAO system, based on the idea of recording an Interaction Matrix between the detector of the camera toward which the aberrations must be corrected and the DM(s) making the correction. While fast and able to provide rather good levels of compensation, it is not intrinsically tomographic and suffers from a number of limitations: the SNR of the measurements, the modal base to use and the filtering during the inversion of the IM and the number of measurement points. Another approach, is to perform classical phase diversity to retrieve the phase in each direction and then project back the measured phases onto several layers of aberrations conjugated at several positions along the optical path.

In this paper, we present an instrically tomographic phase diversity approach, combining all the information from various targets dispatched over the FoV to fit a regularized multi-layer model of static aberrations. Such an approach can be more resilient to noise and does not require the inversion and subsequent filtering of an identification matrix.

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2 Image formation model

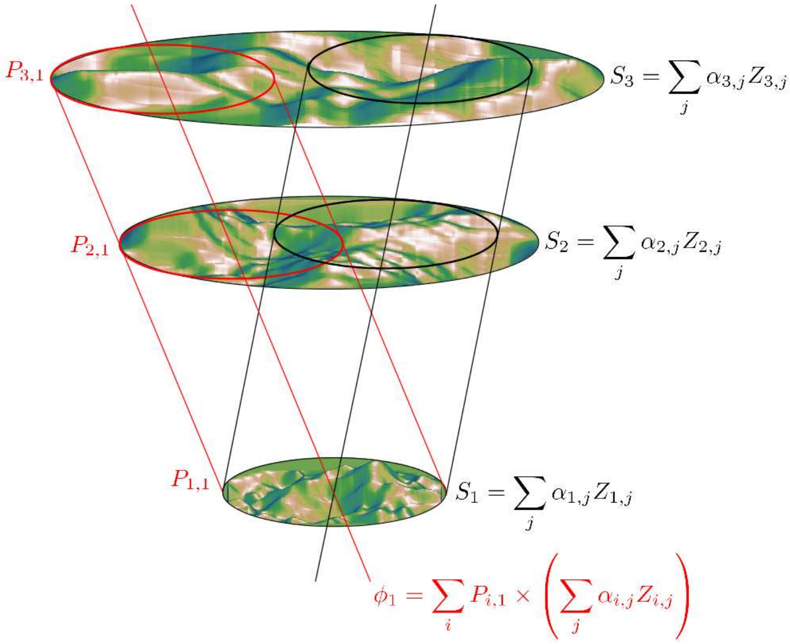

Fig. 1. Schematic view of the phase model in the image formation process described in section 2.

MCAO systems usually include several wavefront correctors conjugated at altitudes characterizing the atmospheric turbulence. In this scheme, the point spread function (PSF) varies across the FoV, depending on the varying combination of aberrations in each directions. In a given direction k, the image is given as the sampled convolution of the object o and the PSF obtained in this direction h_k plus an additive noise n:

$$i_k(\mathbf{r}) = (h_k \otimes o)(\mathbf{r}) + n(\mathbf{r}) \tag{1}$$

where $\otimes$ is the convolution operator and $\mathbf{r}$ is a two-dimensional vector in the image plane. Under the near-field approximation, the point spread function is given by:

$$h_k(\mathbf{r}) = ||\mathbf{FT}^{-1}\{P(\mathbf{u}).exp[j\phi_k(\mathbf{u})]\}||^2 \tag{2}$$

where ϕ_k is the phase in the direction k, $\mathbf{FT}^{-1}$ is the inverse Fourier transform, P the binary aperture function and $\mathbf{u}$ a two-dimensional vector in the pupil plane.

Following the above assumption, the phase in the direction k is given by:

$$\phi_k(\mathbf{u}) = \sum_l \phi_l(\mathbf{u}).P_{k,l}(\mathbf{u}) \tag{3}$$

where ϕ_l is the phase for a given layer at altitude l and $P_{k,l}$ is the aperture function projected at altitude l for the direction k.

Additionally, the phase at altitude l can be expanded on a finite set of modes, for instance, the Zernike polynomials:

$$\phi_l(\mathbf{u}) = \sum_m \alpha_{l,m}.Z_{l,m}(\mathbf{u}) \tag{4}$$

Hence the overall phase for a given direction is given by:

$$\phi_k(\mathbf{u}) = \sum_l \left(\sum_m \alpha_{l,m}.Z_{l,m}(\mathbf{u}) \right).P_{k,l}(\mathbf{u}) \tag{5}$$

Phase diversity uses two or more images differing by known aberrations (for instance defocus). Hence, for each direction, we consider one *in focus* image described by its PSF given in equation 2 and one (or several) *diversity* image whose PSF is given by:

$$h_{k,d}(\mathbf{r}) = ||\mathbf{FT}^{-1}\{P(\mathbf{u}).exp[j(\phi_k + \phi_d)(\mathbf{u})]\}||^2 \tag{6}$$

where ϕ_d is the known aberration introduced in the phase. It is easier in terms of optical setup as well as mathematical treatment to introduce the known aberration on the ground layer so that it is the same for all directions.

A schematic description of this model is provided in Figure 1.

3 Image and OTF-based approaches

Using this image formation model, it is possible to extend the classical 2D phase diversity algorithm to a 3D approach combining the images of several sources across the FoV. Such a method is intrinsically more over-constrained and robust than classical phase diversity in individual direction followed by back projection on discrete layers when the number of positions / images is larger than the number of phase planes in the model. This reduces the possibility to be trapped in a local minima when minimizing a fidelity criterion. In the following, we present to approaches for implementing such a tomographic reconstruction: a *classical* approach using focal plane images and an *alternative* approach in the Fourier domain. They both give similar results.

3.1 PRAy: an image-based algorithm

Because there is a nonlinear relationship between the images and the phase, the retrieval of the latter has to be done, for instance, through the minimization of a dedicated likelihood criterion measuring the fidelity of the above model to the data. Such a method relies on some assumptions regarding the noise statistics. In the case of near-IR imaging, it is reasonable to assume the noise as non stationary Gaussian as proposed by [8]. Moreover, because the object is considered point-like or with a known shape and because we can work at a rather high signal-to-noise ratio (SNR), we considered the simplest likelihood criterion with no penalty term:

$$J(\{\phi_l\}) = \sum_k \left(\frac{||i_k - h_k \otimes o||^2}{\sigma_k^2} + \frac{\left\|i_{k,d} - h_{k,d} \otimes o\right\|^2}{\sigma_{k,d}^2} \right) \tag{7}$$

where the k index stands for the targets in the field. Such an approach naturally provides the ability to mitigate the impact of noise on the image, using a non-stationary version of the noise variance $\sigma_{k,d}^2$ estimated directly on the data as a clipped version of the image for levels above the detector noise variance. This allows to take into account the variable SNR across the diversity images for each source by properly weighting each individual fidelity term.

We implemented such an approach under the form a Yorick program codenamed PRAy[3] and using a gradient descent method: the OptimPack Library[9], for the log-likelihood minimization. The latter is based on a Variable-Metric with Limited Memory (VMLM) approach with optional bound constraints on the parameters. It requires an analytical version of the criterion gradients but provides a fast and robust implementation of multiple variable functions minimization.

The main features of this approach are:

- Classical or tomographic phase diversity

– Zonal (influence functions), modal (Zernike, Karhunen-Loeve) or pixel basis for the reconstructed phase planes
– 2D Gaussian kernel for the object (can be point-like as well)
– It can fit focus scale in diversity sequence and differential tip-tilt between stars

3.2 OPRA: an OTF-based algorithm

The Fourier domain provides another way of viewing the problem. Instead of minimizing the fidelity between the model and the data in the image plane, a similar estimation can be made in the OTF plane corresponding to the Fourier transform of equation 6. This approach, originally proposed by Francois Rigaut as been implemented in the OPRA code[5]. It relies on a least-square estimation of the various parameters of the model using a Levenberg-Marquardt algorithm for the minimization of a fidelity criterion similar to equation 7 but in the Fourier domain.

Such a method is based on finite difference for the gradient descent and is thus slower but it allows to introduce arbitrary parameters in the model such as image wavelength. The main features of this approach are:

– several basis of modes to reconstruct the phase: Disk harmonic (the default), Karhuenen-Loeve or Zernike
– optional yao[6] framework integration, which allows the user to define more complex conditions/systems.
– can fit focus scale in diversity sequence and differential tip-tilt between stars and image intensity
– can fit image pixel scale and lambda
– includes a 2D Gaussian kernel to take into account vibrations

4 Validation on simulated data

As a first order validation, this approach can be tested on simulated data. We performed such a validation on a simple 2 layers model @ 0 et 9 km, with 100 zernike per layer, and a phase RMS of 200-300nm on each layer. To study the impact of meta-pupils overlapping on altitude layers, we studied two cases with respectively 4 and 8 stars dispatched over a 90” x 90” FoV. The results are presented in Figure 2. Various defocus distance for the diversity have been tested form 200 to 800nm (corresponding to the various plots colors) and the plots represent various level of noise. The error is estimated as the residual phase (incident minus reconstructed) RMS over the incident phase RMS.

These results show that:

– the largest defocus values give better results
– at poor SNR the performance are very low especially for the altitude layer.
– pupil coverage is very important to provide reasonable performance for the altitude layer
– a poor reconstruction of the altitude layer seems to impact the ground layer reconstruction especially when few targets are available as shown by the performance of the 4 stars case

The tomographic phase diversity approach has also been validated on a 3 layers model (respectively at 0, 4.5 and 9 km) using 8 stars and the largest defocus distance. Like in the first set of simulations, the incident aberrations are modeled using 100 Zernike polynomials and leading to a phase RMS of about 200nm per layer. In figure 3, we show the results for respectively low and high SNR in the images. It demonstrates that:

– the ground layer reconstruction is less affected by poor SNR because the information is averaged on a large number of stars
– a good reconstruction level can be achieved on multiple layers at high SNR

A more thorough analysis is required but these first results are sufficiently encouraging to start working on real data. The following section presents some preliminary reconstruction results on GeMS.

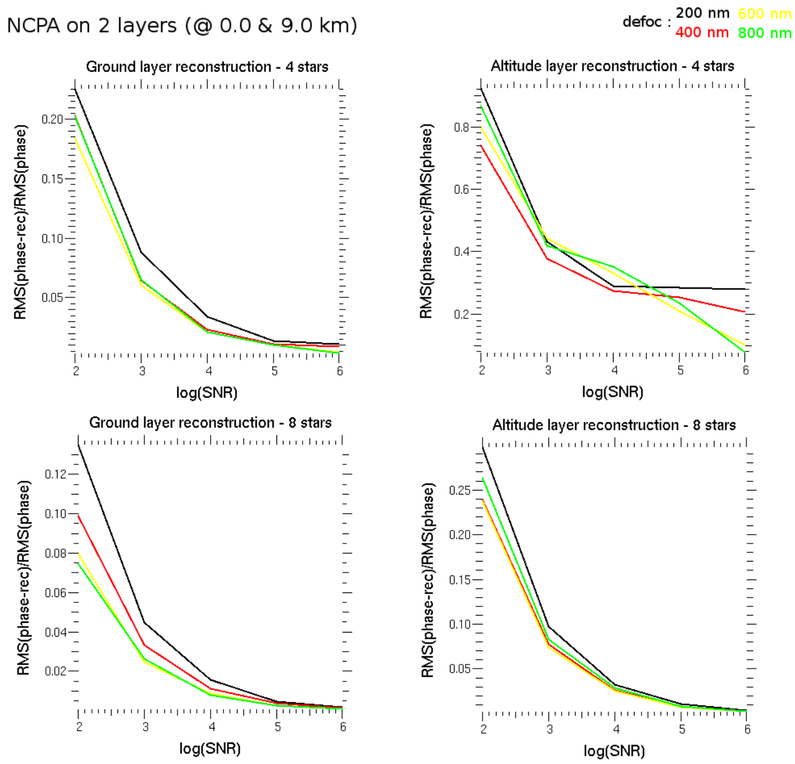

Fig. 2. Reconstruction errors for a 2 layers model for different number of targets (top: 4, bottom: 8) and various noise level. Each color corresponds to the value of the defocus used for diversity.

5 First results on GeMS

GeMS is the Gemini Multi-conjugate adaptive optics System. It is equipped with three deformable mirrors totaling 917 actuators (684 active, 233 extrapolated), conjugated optically to 0, 4.5 and 9 km above the telescope. GSAOI (a $4k^2$ NIR imager), providing 85 arcsec FoV images with 20 mas per pixel[4] is one of the dedicated instruments. It has recently undergone on-sky commissioning and first spectacular results have been presented by Rigaut et al. in this conference[7].

We first had a chance to validate the approach on a series of images obtained during GeMS initial phases of commissioning in March 2011. A series of images with one in focus and 2 out of focus positions respectively at 400 and -400nm have been acquired for 19 calibration sources covering GSAOI FoV. We used a model of the GeMS mirrors as a zonal basis for the reconstruction on each layer with additional tip-tilt and focus on the ground layer. The results are displayed in Figure 4 for various stars located at different position in the field. The result of the reconstruction in compared to the input data in each case. The stars repartition is shown in the central part of the figure. See the legend for more details on the position of each example. The reconstructed phases on the 3 GeMS mirrors has also been displayed. They show various level of granularity consistent with the various actuators densities at each altitude (respectively 17, 20 and 12 in the pupil diameter).

Figure 4 demonstrates the efficiency of the tomographic phase diversity approach to provide an accurate model for the images of various calibration sources dispatched across the FoV of GSAOI and a 3 layers model for the corresponding aberrations to be corrected.

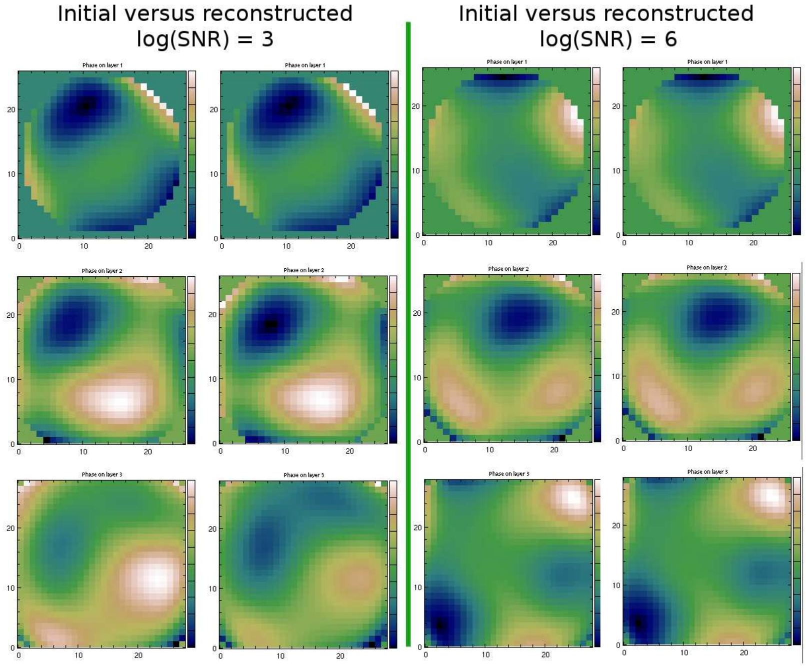

Fig. 3. Initial versus reconstructed phase for 3 layers (at 0, 4.5 and 9km) and in the case of respectively low (left) and high (right) SNR. In each panel the incident phase is displayed left and the reconstructed one is right.

6 Future work

Since March 2011 GeMS has lost one DM and is now working with 2 DMs conjugated at 0 and 9 km (see [7]). The GeMS team was nonetheless able to reach H band Strehl ratio on the calibration sources of 88% averaged over the entire output in two iterations, the tomographic phase diversity process using 24 targets in the FoV and 2 output DMs with about 100 modes each. Additional tests has been led by the GeMS team, using this technique on-sky for the estimation static residuals in close loop (see [7] for details), showing the great potentiel for this approach. A more thorough study is now necessary to estimate and optimize the performance under various conditions.

References

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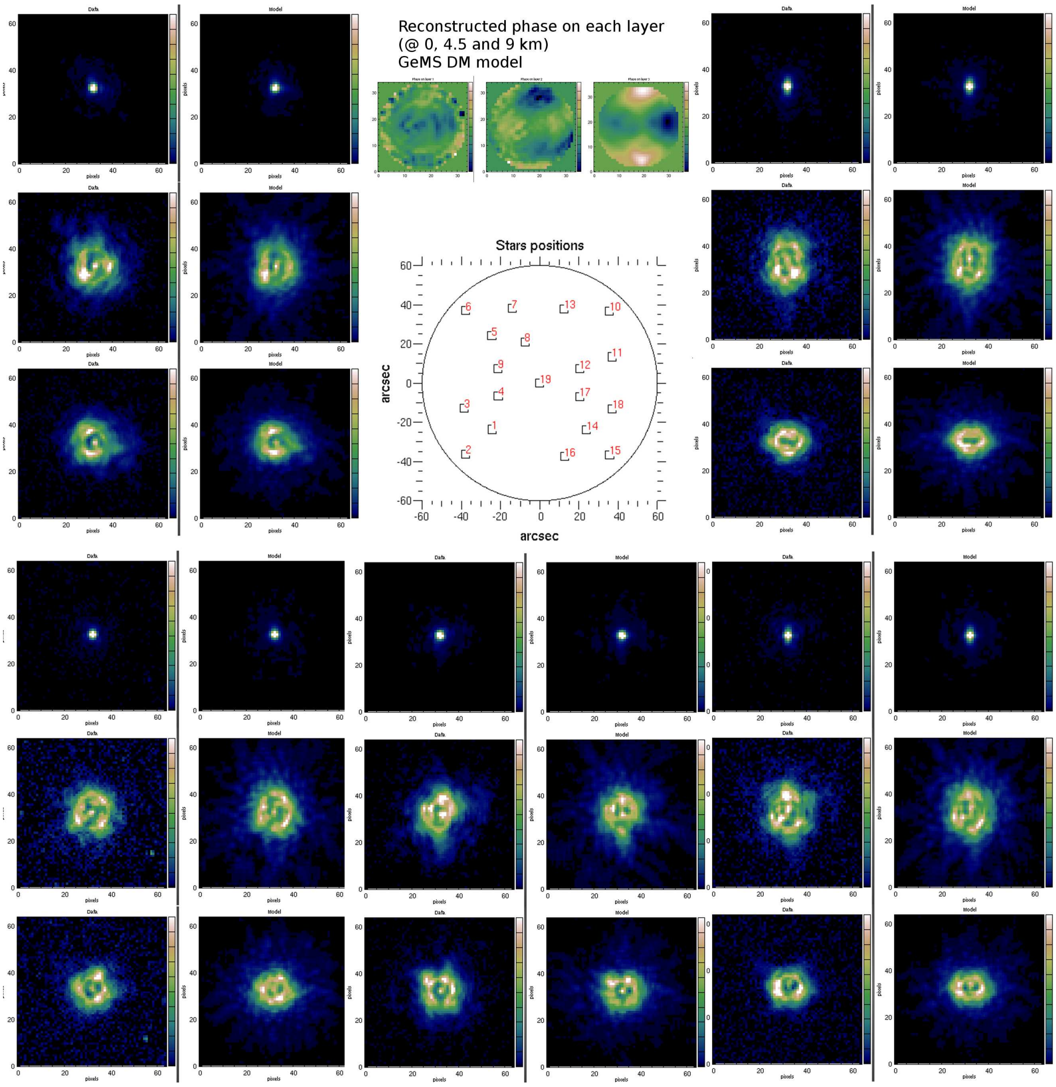

Fig. 4. Reconstruction results for various calibration sources in the FoV, form top left to bottom right : 1, 12, 6, 19 and 16. In each case the in focus and both defocused images are shown from top to bottom, left in the measure data and right in the reconstructed version.