The Pyramid Wavefront Sensor with Extended Reference Source

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Abstract. During the LBT FLAO commissioning, the Pyramid Wave-Front Sensor demonstrated on sky its potential as component of a NGS SCAO system. These results confirmed the expected P-WFS sensitivity in presence of a point-like guide source. The performances of the P-WFS using an extended object as reference in astronomical applications have not yet been deeply investigated. In this work we present some preliminary investigation on this topic. We will show here: a first experimental result on the sensitivity obtained in laboratory with the LBT FLAO system\#2 using extended reference sources of diameter (projected on-sky) up to 1.6" and some consideration on the effect of a vertical extension of the source as in the case of a Na LGS.

1 Introduction

The Adaptive Optics (AO) systems for the next generation of ground based telescopes (aka ELT) will widely employ the Laser Guide Star (LGS) technology \cite{1} \cite{2} \cite{3}. At present, the designs of all these systems employ Shack-Hartmann (SH) WFSs, being this one the sensor successfully used in all the LGS AO system of the 8–10m class of telescopes. Recently the Pyramid WFS (P-WFS) demonstrated on sky its enhanced sensitivity \cite{4} (with respect to the SH-WFS) in the natural guide star AO system of LBT called First Light Adaptive Optics (FLAO) system. While the properties of the P-WFS operated on a natural guide star have been studied in the past and now proven on sky, the behavior of this WFS operated on a LGS still needs to be investigated. Preliminary simulations have been performed in the past \cite{5} and in this paper we will present the first experimental data obtained on a P-WFS operating on an extended source in an AO closed loop, together with some theoretical considerations on the effect of the vertical extension of the Na layer.

2 Closed loop performances using a reference source extended in the focal plane

In this section we report the results we obtained with the (FLAO) system during its testing phase in the Arcetri test tower. These measurements were aimed to measure the P-WFS sensitivity as a function of the source extension in the focal plane. As already demonstrated in the literature, the P-WFS has a higher sensitivity with respect to the Shack-Hartmann WFS (SH-WFS) when it is working in diffraction limited (DL) regime \cite{6} or in partial correction regime \cite{7}. This is mainly due to the fact that the P-WFS uses the full-aperture PSF to sense wave-front (WF) aberrations, while the SH-WFS uses PSFs from sub-apertures corresponding to the lenselts of the SH array. When the reference source is an extended source, its image on the pyramid tip will be always bigger than the diffraction-limited one, so part of the pyramid gain is lost because of its extension. In our knowledge, up to now, no quantitative study or experiments on this topic have been reported in literature.

2.1 The laboratory set-up

We realized the laboratory tests reported in this paper using the setup described in \cite{8}. The main components are the P-WFS and the Adaptive Secondary Mirror (ASM) composing the FLAO system.

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In our tests we used the FLAO2 system that was installed in the Arcetri test tower for the acceptance test operations. The Arcetri test tower is a test bench arranged in a 14m thermalized pipe. The whole setup is reported in Fig. 1. We used as reference source the calibration source of the FLAO system: a compact optical system that provides a telecentric F15 beam re-imaging an optical fiber with unitary magnification. During the FLAO calibration and test we used a fiber of 10µm of core diameter that results in an unresolved object at the P-WFS operating wavelength. During the experiment described below we used fibers with a wider core in order to generate an extended reference source.

In Sect. 2.2 we will show two IR images acquired with the InfraRed Test Camera (IRTC) [9]. This camera is located at the focus of the system close to the P-WFS where a dichroic reflects the the visible light towards the P-WFS while the IR (λ > 1.0µm) is transmitted to the infrared camera. The IRTC and the dichroic are not represented in Fig. 1.

2.2 Closed loop with different source dimensions

In this section we will compare closed loop performances obtained with sources of different sizes. As done for the FLAO acceptance test, we simulated the atmospheric disturbance using the ASM itself [8]. In this way, we know the applied disturbance with an error of few nm and, recording the actuator positions in closed loop, we know step by step the effective AO residual with the same accuracy. In all the data reported in this paper, the disturbance we applied was simulating a seeing of 0.8" with a wind of 15m/s.

We used the modal residual of the Adaptive Optics correction as metrics for the performances because the only source available on the focal plane was the reference source and its extension made complex and unreliable the Strehl Ratio (SR) estimation. In order to give a reference for the correction in terms of SR, we selected two cases of closed loop on a diffraction limited source at different flux: \( m_R = 9.6 \) and \( m_R = 10.9 \), presenting a SR in H band of 0.85 and 0.67 respectively. We measured those SRs on the images recorded on the IRTC and reported in the boxes of Fig. 2. Then we reported the modal residual for these two cases together with those obtained with the extended sources, so that their residuals can be directly compared.

As extended sources we selected fibers with cores of diameter: 120, 600 and 1000µm at the \( F_{15} \) focal plane of the system that correspond to an angular diameter projected on-sky of 0.2", 1.0" and 1.6"
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In the FLAO case, the WFS works with light in between 0.6 and 0.9$\mu$m, while $D = 8.2m$. Taking 0.75$\mu$m as effective value of $\lambda$ we obtain $\alpha_\lambda = 0.23''$ in good agreement with the source apparent angle estimated as $\alpha_\lambda = 0.20''$. In the case of the wider extended sources we tested, unfortunately this effect can not be verified because to $\alpha_\lambda > 0.6''$ correspond modes > 550 that are not corrected in our closed loop.

More in general, we can note in Fig. 2 that the P-WFS loses sensitivity progressively with the increasing of the reference source extension. However, this reduction in sensitivity is not dramatic being comparable to the loss of ~ 1 magnitude in reference star brightness. Of course here we do not take into account the cone effect because is out of the scope of the document, being independent of the WFS nature.

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**Fig. 2.** (Color on-line) The modal wavefront variance of the injected disturbance (red line) and in closed loop (all the other line colors). The black represents the closed loop with DL reference source, in the two boxes are reported the H band PSFs of these two cases. The green, light blue and dark blue represent the variances obtained in closed loop using source of 0.2", 1.0" and 1.6" respectively.

respectively. We set the integrated magnitude of the reference source around $m_R = 9.4$ in order to work with a flux comparable with the return of available laser guide stars [10]. We performed the correction at 1kHz of frame rate and at the maximum spatial sampling available on FLAO that corresponds to 30 sub-apertures on the pupil diameter. In this first part of the experiment we optimized the integrator gain of the AO loop using only two independent values: the first one for tip and tilt, the second for all the remaining modes.

The result obtained with the fiber of 0.2” shows an interesting behavior. Comparing the correction obtained in this case with respect to the DL one at $m_R = 9.6$, we can notice that the correction is slightly worst for modes lower than ~ 80, while for the higher ones we measured the same correction as obtained with the DL source. This can be explained by considering the extension of the source acting, as first approximation, like the tip-tilt modulation. We can think to a DL modulated source an extended annular source with thickness equivalent to the DL FWHM and radius equal to the one of the modulation [11]. When we have an extended source, the light is distributed on a wider area and the sensitivity of the WFS is reduced but its effect is not the same for all the corrected modes. As in the case of the modulation, the sensitivity reduction acts only on the modes having a radial order that corresponds to angles smaller than the applied modulation [12] or, in this case, to the size of the extended object. In the case of the 0.2” source, we mentioned that the sensitivity reduction acts up to mode ~ 80. The 78th mode closes the radial order $n = 11$, corresponding to an angle on sky of $\alpha = (n + 1) \lambda / D$, being $\lambda$ the WFS working wavelength and $D$ the pupil diameter projected on sky.

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2.3 Noise propagation with a source of 1.6"

During the test on the system, we investigated further the case of the reference source having a diameter of 1.6". We closed the loop changing the source brightness and we compared the modal residuals with those obtained with the DL one at comparable light fluxes. In Fig. 3 we can notice how the curves corresponding to $m_R = 8.5$ for the extended source and $m_R = 9.4$ for the DL one are substantially superimposed for the modes < 100; the same happens for the curves related to $m_R = 9.4$-extended and $m_R = 10.9$-DL. This tells us that for modes < 100 the loss in sensitivity due to the source extension is comparable to the loss in flux of 1 magnitude.

From the same sets of data analyzed before, we can compute the noise propagation coefficients for the extended and DL source cases. Let us consider now the average residual for the modes in between 200 and 250 for each of the curves shown in Fig. 3. If we are in a photon noise limited regime the residual should scale as $\sigma^2 \propto 1/N_p$, where $\sigma^2$ is the residual variance and $N_p$ is the number of photons detected per sub-aperture on the WFS frame. We can write this relationship as $\sigma = k/\sqrt{N_p}$ where $k$ is defined as the photon noise propagation. In Fig. 4 we report the plot of $\sigma$ as a function of $1/\sqrt{N_p}$ for the case of the extended source and the DL one. The expected linear relationship is in good agreement with the data (see Fig. 4), confirming the hypothesis of being in photon noise limited regime. The values of $k$ we found are 62 and 30($nm/\sqrt{ph}$) for the extended and DL sources respectively.

We repeated then the same exercise for each mode in the range 10 – 250 founding again that the linear relationship is quite well respected. The values of $k$ for all the modes are reported in Fig. 5 for both sources. Let us represent as $k_E(m)$ and $k_D(m)$ the noise propagation coefficient computed on the mode $m$ for the extended and DL source respectively. From the plots in Fig. 5 we can say that the ratio $R(m) = k_E(m)/k_D(m)$ is always in the range 1.5 – 3. The value of $R$ results to be lower than expected. It can be demonstrated that the expected sensitivity of the P-WFS for an extended source (with a constant flux distribution on it surface) can be scaled to an equivalent radius of modulation (for a DL source) of half its diameter. Because in the diffraction limited case we applied a modulation of 0.12" in diameter, the expected value is $R = 1.6"/(2 \cdot 0.12") = 6.7$ against the measured $R_m$ in between 2 and 3. The origin of this discrepancy still requires further investigations to be better understood.
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**Fig. 4.** (Color on-line) The diamonds represent the measured $\sigma$ averaged on modes 200 – 250 at different flux levels for the DL source case (black) and the 1.6” source (blue). The lines represent the two linear fittings that indicate a slope of 30.0 and 62.5 nm $\sqrt{ph}$ for DL and extended source respectively.

**Fig. 5.** (Color on-line) As in Fig. 5, the noise propagation coefficient $k$ has been computed for all the modes in the range 10 – 250. Here we report the value of $k$ vs. the mode number. Again the color black and blue indicate the case of DL and extended source.

3 The Na layer thickness effect: a theoretical estimation

Let us consider now the extension of the source in the line of sight. As it is well known [13], the Na laser generates a source that has a finite extension along the optical axis and this generates on the Shack-Hartmann (SH) WFS the so called spot elongation effect. This elongation unfortunately changes in shape and in dimension because of the Na layer variability, generating one of the main
limitations of the LGS WFSs for the ELTs. Due to the relevance of this topic we decided to provide an estimation of the effect on the P-WFS with some simple theoretical considerations.

3.1 The layer thickness as seen by the pyramid

Let us consider the Na layer at an average distance $h_0$, extended from $h_L$ to $h_H$, and a telescope having a pupil of diameter $D$ (see Fig. 6) imaging the layer at distance $h_0$. Shooting the laser from the center of the pupil of the telescope, the wavefront error on the entrance pupil due to imaging out-of-focus layer at $\Delta h$ distance from $h_0$ is given by:

$$WF(r) = \frac{\Delta h}{8\left(\frac{h_0}{D}\right)^2} \left(\frac{r}{D/2}\right)^2 = \frac{\Delta h}{2h_0^2}r^2$$

where $r$ is the distance from the center on the entrance pupil of the telescope.

The on-sky tilt $\theta(r)$, corresponding to the local derivative of the wavefront error at radial distance $r$ is given by:

$$\theta(r) = \frac{\Delta h}{h_0}r$$

Because we have a continuous distribution of focus error, each sub-aperture (SA) senses a continuous distribution of tilts like it happens in a tilt modulated pyramid WFS. The main difference is that, in the case of the LGS, the tilt modulation experienced by each sub-aperture $\theta$ varies with $r$ and is in the radial direction only.

We can estimate the amount of this radial modulation evaluating $\theta(r)$ on a sub-aperture at the pupil edge ($r = D/2$) and close to the central obstruction ($r = \epsilon D/2$), where $\epsilon$ is the obscuration ratio. Considering $\Delta h = \pm 5$km, in the case $D = 8m$, taking $\epsilon = 0.15$, we get $\theta(\epsilon D/2) = \pm 0.08^\circ$ and $\theta(D/2) = \pm 0.51^\circ$, while for $D = 42m$ and $\epsilon = 0.3$, we get $\theta(\epsilon D/2) = \pm 0.80^\circ$ and $\theta(D/2) = \pm 2.7^\circ$.

3.2 Pyramid and Shack-Hartmann comparison

As we have just found in the previous section, the Na layer thickness induces on the P-WFS a loss of sensitivity in the radial direction that increases with the distance from the pupil center (in the case of LGS launch from the center of the telescope pupil). This is the same effect caused by the spot elongation on the SH-WFS. In literature [13] we can find that the estimated spot elongation seen by a SH-WFS has typical dimension comparable with those found for $\theta(r)$ on the P-WFS. So we can
expect that the sensitivity of P-WFS and SH-WFS result comparable. However there is still a relevant difference in the features of the two WFSs: the Field of View (FoV) of the P-WFS is determined by the field stop usually placed in front of the pyramid tip, while for the SH it is determined by the FoV of the single lenslet on the detector. As a consequence, the SH FoV is determined by design and has to take into account also the bad conditions of the Na layer extending its dimensions. In the literature [14] [15] for the E-ELT we find SH-WFS FoV values of 9” – 15”. This big FoV is usually associated with a large sampling in the focal plane in order to keep the linearity of the SHWFS using the weighted center of gravity algorithm for the signal computation. This sampling is considered to be in between 0.5 – 1”/pixel driving the number of pixels per SA from 9x9 to 25x25. Considering 84 sub-aperture on the 42m pupil diameter, the SH-WFS will require a detector having from 800 to 2000 pixels on a side. This kind of device, with low noise and high readout speed, is not available today and present a serious challenge for the current technology. On the other hand, the P-WFS can change its FoV with a simple adjustable iris placed in front of the pyramid and, much more important, will require only 2x2 pixels per SA regardless of the chosen FoV. That means that a P-WFS with the same sampling (84 SA on the pupil diameter) could be realized today with a commercial EMCCD chip of 240x240 pixels, like the OCam camera [16].

4 Conclusion

The experimental data presented in this paper show that an extended source of 1.6” (projected on sky) will reduce the sensitivity of the P-WFS of about a factor 2 with respect to the one obtained with a DL source modulated 0.12” in diameter. We can compare this result with the sensitivity of a SH-WFS considering the wavefront residual theoretical ratio. As reported in literature [6], we have in the diffraction limited case, without modulation, $\sigma_{\text{DL}}^2 = (r_0/D)^2 \sigma_{\text{SH}}^2$ being $r_0$ the Fried parameter at the WFS wavelength, $\sigma_{\text{DL}}^2$ and $\sigma_{\text{SH}}^2$ the residual phase variance of the P-WFS and of the SH-WFS in DL conditions respectively. For the DL case with 120” of modulation, as in our experiment, we obtain the residual variance $\sigma_{\text{PM}}^2 = (6r_0/D)^2 \sigma_{\text{SH}}^2$, because the modulation we applied corresponds to 6 times the DL spot size. In Sect. 2.3 we found that $\sigma_{\text{ES}}^2 \sim 4\sigma_{\text{PM}}^2$, where $\sigma_{\text{ES}}^2$ is the residual variance obtained with the source of 1.6”. This leads to $\sigma_{\text{ES}}^2 \sim (12r_0/D)^2 \sigma_{\text{SH}}^2$. Having $D = 8$m and $r_0 = 0.2$m, we obtain $\sigma_{\text{ES}}^2 \sim 0.1\sigma_{\text{SH}}^2$, while, considering the case of the E-ELT with $D = 42$m, we have $\sigma_{\text{ES}}^2 \sim 3 \cdot 10^{-3}\sigma_{\text{SH}}^2$. The sensitivity estimation presented in the paper is preliminary, requiring to be confirmed, but the result still indicates a clear gain of the P-WFS with respect to the SH-WFS also in the case of an extended reference source. This is especially true in the case of an ELT. In fact, if the gain in sensitivity may not allow a better correction because of the cone effect, it will still translate in a reduction of the LGS power required to provide a certain level of correction.
In the second part of the paper we analyzed the effect of the Na layer thickness. The presented simple considerations show how the sensitivity reduction in this case is comparable with the one experienced by the SH-WFS. However we demonstrated that the P-WFS nature drives to a critical practical advantage in the number of pixel required for the realization of the WFS.

The results presented show that the P-WFS is an attractive alternative to the SH-WFS also for LGS AO systems, especially for in the ELT case. Further investigations are required in order to confirm these results and study other optical configurations as the LGS launched from the telescope pupil edge.

References