The Slope-Oriented Hadamard scheme for in-lab or on-sky interaction matrix calibration

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Abstract. The correct calibration of the interaction matrix affects the performance of an adaptive optics system. In the case of high-order systems, when the number of mirror modes is worth a few thousands, the calibration strategy is critical to reach the maximum interaction matrix quality in the minimum time. This is all the more true for the E-ELT, for which on sky calibration procedures have to be considered. Here, we first build a tractable interaction matrix quality criterion. We then propose the Slope-Oriented Hadamard scheme which optimizes this quality criterion.We demonstrate that for a given level of quality, the calibration time needed using the Slope-Oriented Hadamard method is ten times less than with a classical Hadamard scheme. These analytical and simulation results are confirmed experimentally on the SPHERE XAO system (SAXO).

1 Introduction

The calibration procedure of the interaction matrix consists in sending a set of actuation patterns $\boldsymbol{V}_i$ and to measure the yielded wavefront sensor measurements $\boldsymbol{S}_i$, affected by a measurement noise $\boldsymbol{N}_i$. With $\boldsymbol{V}$, $\boldsymbol{S}$, $\boldsymbol{N}$ the matrices which rows are the $\boldsymbol{V}_i$, $\boldsymbol{S}_i$, $\boldsymbol{N}_i$, we obtain:

$$\boldsymbol{S} = \boldsymbol{D}\boldsymbol{V} + \boldsymbol{N}$$

with $\boldsymbol{D}$ the interaction matrix to identify. The estimated interaction matrix is retrieved by multiplying the measurement matrix $\boldsymbol{S}$ by the pseudo-inverse $\boldsymbol{V}^\dagger$ of $\boldsymbol{V}$: $\hat{\boldsymbol{D}} = S\boldsymbol{V}^\dagger$. The calibration error is given by

$$\Delta\boldsymbol{D} = -\boldsymbol{N}\boldsymbol{V}^\dagger$$

The classical method consists in pushing one actuator at a time with a voltage v, which corresponds to a diagonal matrix $\boldsymbol{V}$:

$$\boldsymbol{V} = v \cdot \mathbf{I}\mathrm{d} \Rightarrow \Delta\boldsymbol{D} = -\boldsymbol{N}/v$$

v has then to be set at the maximum value compatible with the system, *i.e.* without saturating the wavefront sensor nor the deformable mirror.

If one considers only the saturation of the deformable mirror *voltages*, the optimal solution (with respect to a quadratic norm on $\Delta\boldsymbol{D}$) is

$$\boldsymbol{V} = V_{\mathrm{sat}} \cdot \boldsymbol{H}$$

with $\boldsymbol{H}$ a Hadamard matrix, which contains only -1 and 1 values[1]. However, this Hadamard "voltage oriented" actuation scheme is not suited to closed-loop systems for which the wavefront sensor has a much smaller linearity range (suited to measure the *residual* turbulence) than the deformable mirror (suited to correct for the *total* turbulence). We propose here a Hadamard "slope oriented" actuation scheme obtained by considering the saturation of the Shack-Hartmann wavefront sensor slopes.

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We first describe how we achieve such a slope oriented scheme (section 2). Then we compute an interaction matrix calibration error metric, which we use to compare the zonal, voltage-oriented Hadamard and slope-oriented Hadamard strategies in the configuration of SPHERE's extreme adaptive optics system SAXO[2,3](section 3). The results obtained are then confirmed on an end-to-end simulation of SAXO(section 4). Last we report experimental results obtained on SAXO in august 2011(section 5).

2 The "slope oriented" Hadamard scheme

We consider here only the saturation of the wavefront sensor. The most effective calibration patterns are the one that take all the spots at the limits of the linear range of the wavefront sensor, *i.e.* at slopes $s = \pm S_{\max}$. The problem here is then to find a set of actuation patterns $\boldsymbol{V}$ such that $\boldsymbol{DV}$ contains $\pm S_{\max}$ values. $\boldsymbol{V}$ shall also have a rank superior or equal to the number of actuators, so that the whole deformable mirror shape space is spanned (this ensures $\boldsymbol{V}\boldsymbol{V}^{\dagger} = \mathbf{I}\mathrm{d}$). The way we find such a $\boldsymbol{V}$ consists in three steps:

1. obtain a first rough estimate of $\boldsymbol{D}_0$ (a synthetic matrix can be used);
2. solve for $\boldsymbol{DV} = S_{\max}\boldsymbol{H}$, with $\boldsymbol{H}$ a Hadamard matrix ($S_{\max}\boldsymbol{H}$ contains only $\pm S_{\max}$ values). This is done using $\boldsymbol{D}_0$:$\boldsymbol{V}_0 = \boldsymbol{D}_0^{\dagger} \cdot S_{\max}\boldsymbol{H}$;
3. perform a linear iterative minimization of $\sum_{i,j} \left| \left| [\boldsymbol{D}_0\boldsymbol{V}]_{i,j} \right|^2 - S_{\max}^2 \right|^2$ with respect to $\boldsymbol{V}$, starting from $\boldsymbol{V}_0$.

The result is not optimal. There may exist a strategy to find $\boldsymbol{V}$ with a rank superior or equal to the number of actuators, yielding the lowest interaction matrix calibration error while saturating neither the deformable mirror nor the wavefront sensor. However, the strategy we have described here provides a good approximation of this optimal solution.

We have used this strategy on the SAXO case. The histograms of the slopes $\boldsymbol{DV}$ obtained during the interaction matrix calibration for the zonal, voltage-oriented and slope oriented hadamard strategies are shown in figure 1.

We see that the slope-oriented strategy yields the most energetic slope patterns. The calibration error propagated in the Adaptive Optics closed loop should therefore be lower.

3 A tractable metric for the Interaction matrix error

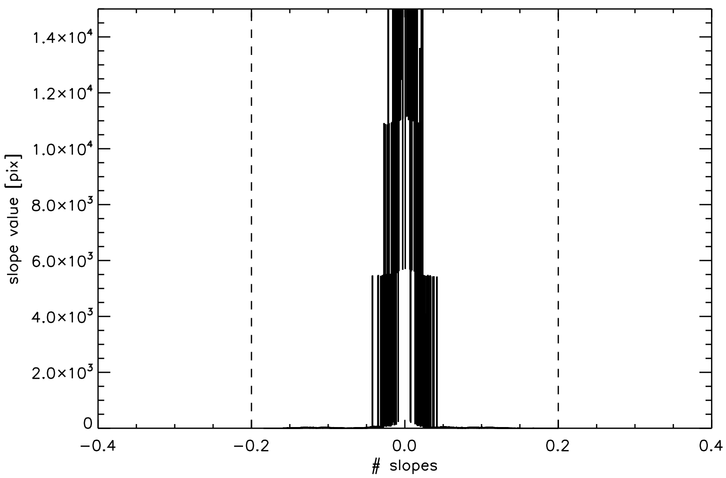
We have previously established a metric Q on the interaction matrix error that conveys the induced closed loop performance loss [4]:

$$Q(\varDelta\boldsymbol{D}) = \|\varDelta\boldsymbol{D}\|^2_{\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}}\,, \text{ with } \|\boldsymbol{X}\|^2_{\boldsymbol{W}} \stackrel{\Delta}{=} \mathrm{Trace}\left[\boldsymbol{X}\boldsymbol{W}^{-1}\boldsymbol{X}^{\mathrm{T}}\right]$$

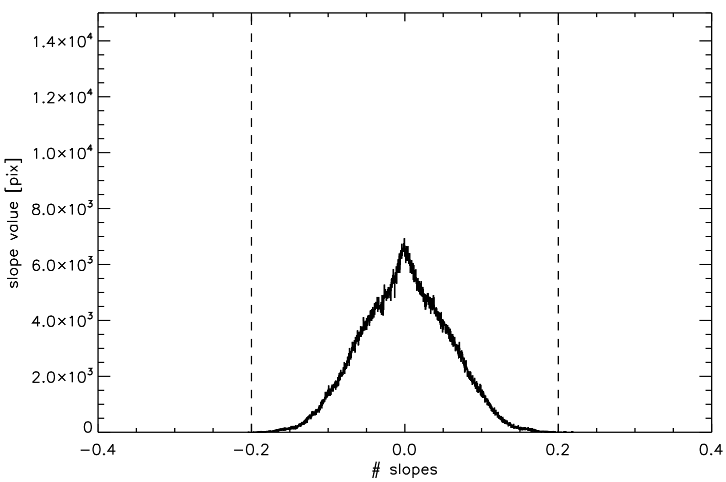
. The value obtained has no dimension whereas the usually used quadratic norm $\|\varDelta\boldsymbol{D}\|^2$ is in slope per volt. Moreover, the $\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}$ weighting propagates the error in the system, allowing to assess the impact of the interaction matrix error in closed-loop on the residual phase variance (see paper to come for precisions[5]).

Using this metric, we have plotted the error $Q(\varDelta\boldsymbol{D})$ as a function of the wavefront sensor error during the calibration, expressed in centroid error variance (fig. 2)

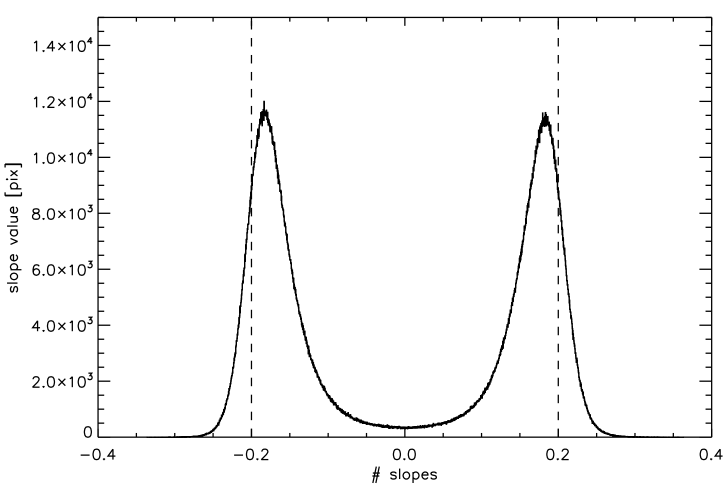
For a very low noise during calibration, the error is due to the non-linearities of the wavefront sensor. This error is approximately the same for the three strategies, which means that all three have bee tuned correctly not to saturate de wavefront sensor. The high noise regime shows the gain to use the slope oriented strategy. A $Q = 0.2$ interaction matrix can be obtained with a noise 100 times greater for the voltage oriented strategy than for the zonal



Zonal



Voltage-oriented Hadamard



Slope-oriented Hadamard

Fig. 1: histograms of the slopes $\boldsymbol{DV}$ obtained during the interaction matrix calibration for the zonal, voltage-oriented and slope oriented hadamard strategies. Dashed lines correspond to $\pm S_{\max}$.

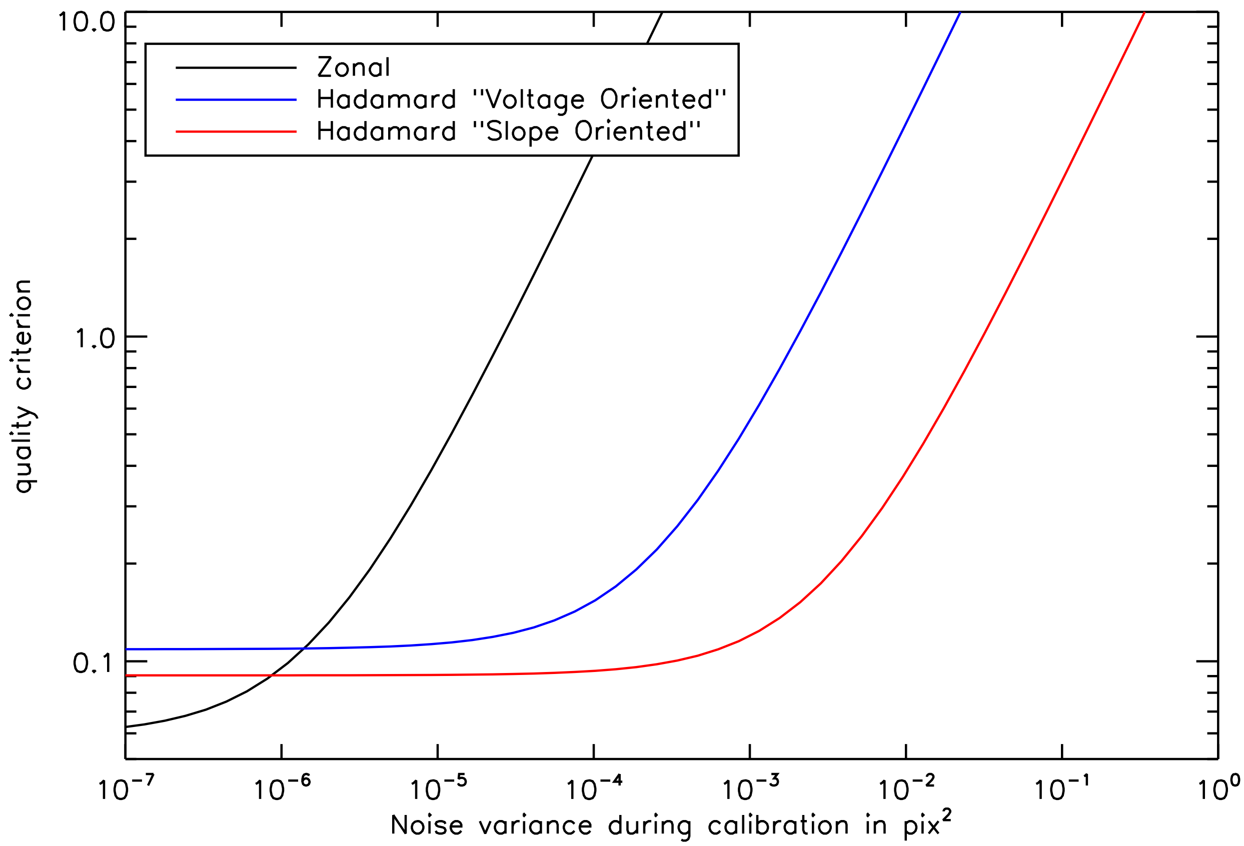


Fig. 2: Interaction matrix calibration error $Q(\Delta \boldsymbol{D})$ as a function of the wavefront sensor error during the calibration, expressed in centroid error variance

strategy. Another factor 10 is gained by using the slope-oriented strategy. In photon noise regime,the centroid error variance is proportionnal to the integration time. In other words, the same quality can be obtained with the slope-oriented strategy in 10 seconds as with the voltage oriented strategy in 1.5 minutes or with the zonal strategy in 2 hours.

4 Closed-loop end-to-end simulation results

In order to confirm that our metric acurately conveys the induced error in closed loop, comprehensive simulations on an end-to-end simulator of SAXO[2,3] have been conducted with the interaction matrices obtained for the three strategies, with various levels of noise on the Shack-Hartmann wavefront sensor during the interaction matrix calibration. Figure 3 plots the residual phase variance obtained as a function of the wavefront sensor noise during the interaction matrix calibration. The residual phase variance level in the low calibration noise regime corresponds to the other terms of the error budget (temporal error, aliasing and fitting errors, the noise error is negligible in this case as we simulated a high flux guide star). We see that the curves present a high increase at the same level as in figure 2. It confirms that our

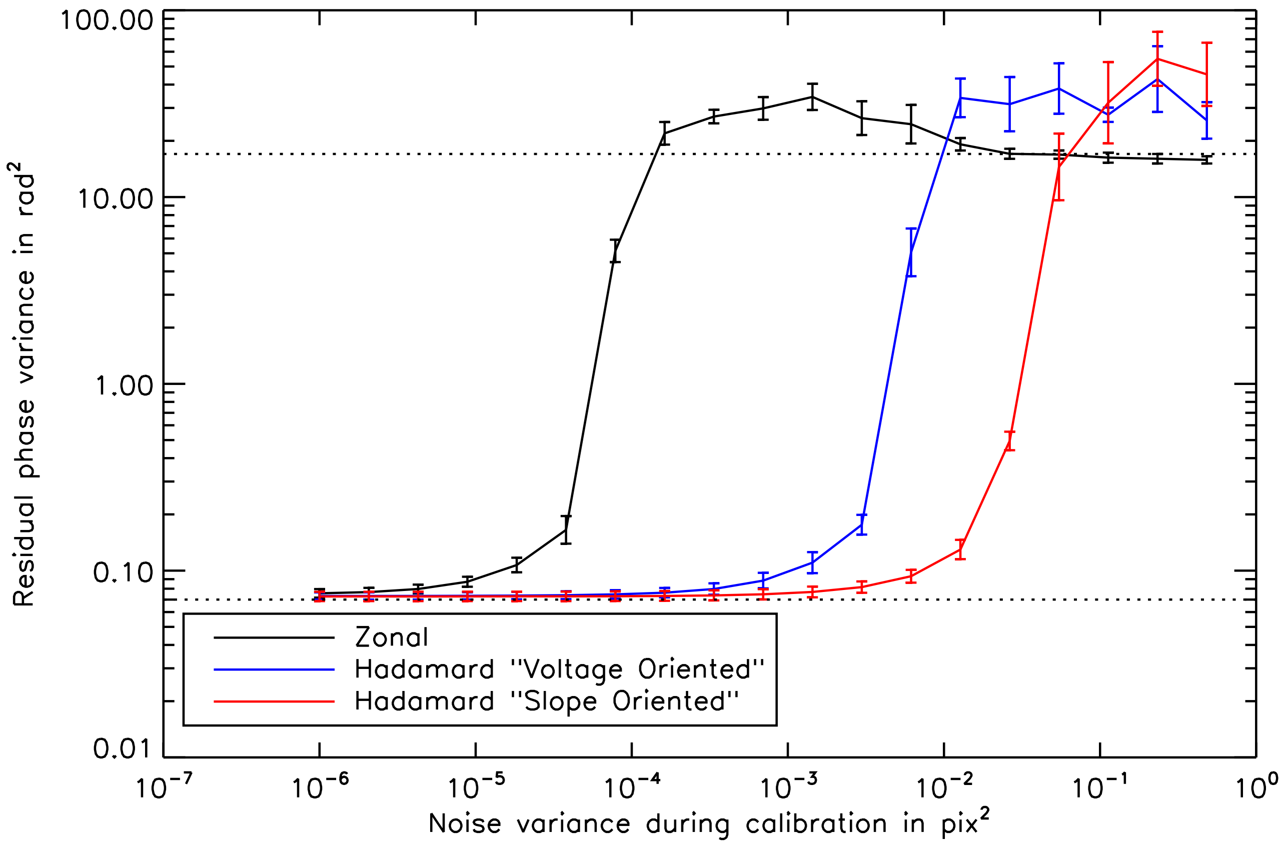


Fig. 3: Residual phase variance obtained as a function of the wavefront sensor noise during the interaction matrix calibration

metric is suited to predict the closed loop residual phase error budget term due to interaction matrix miscalibration.

5 Experimental validation

We have acquired three interaction matrices on the SAXO bench at Observatoire de Paris during August 2011, with the three strategies, and considering a 90 seconds calibration time for each matrix. The three interaction matrices obtained are shown in figure 4, clipped at 1% of their total range in order to visually assess the noise. This emphasizes the major gain to use slope-oriented hadamard strategy. We also report that the loop was closed succesfully during several minutes using the interaction matrix acquired with the slope-oriented hadamard strategy. We witnessed no performance loss compared to the use of a 1.5 hour acquisition zonal strategy calibration matrix.

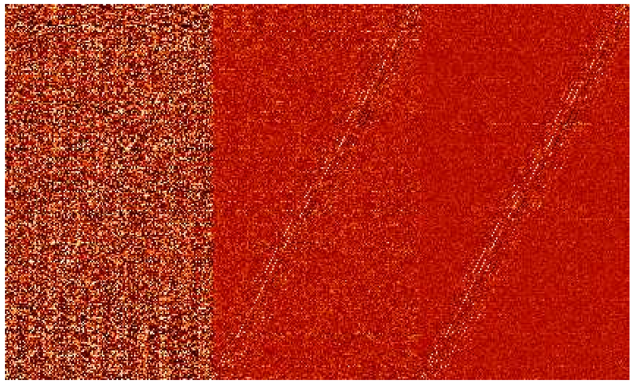


Fig. 4: Experimental SAXO interaction matrices acquired in 90 seconds each. Left: Zonal strategy, Center: Voltage-oriented Hadamard Startegy, Right: Slope-oriented Hadamard strategy

6 Conclusion

We have proposed a tractable metric for the interaction matrix calibration error, and proposed the slope oriented Hadamard calibration strategy which aims at minimizing this metric. We have validated the accuracy of the metric by means of an end-to-end simulation of the SAXO bench, and have demonstrated experimentaly on the saxo bench a 10 fold gain of the slope-oriented Hadamard strategy over the voltage-oriented Hadamard strategy.

References

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