Experimental validation of the linearized focal-plane technique (LIFT)

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Abstract. Laser-assisted adaptive optics (AO) systems should increase dramatically the system sky coverage. Unfortunately, the laser guide star (LGS) wavefront sensing (WFS) principle is insensitive to tip/tilt, and focus measurement is corrupted by the evolution of the sodium concentration in altitude. Additionally, volumic structures of the LGS may induce quasi-static WFS errors. Hence, low-order modes have to be measured separately using faint natural guide stars (NGSs), and a so-called "truth sensor" has to be used to calibrate higher order LGS induced WFS errors. In that framework, we have proposed a new focal-plane WFS concept called the linearized focal-plane technique (LIFT), which allows us to efficiently deal with low-order mode measurement under low flux conditions. We report here our latest experimental results towards a full validation. In particular, we demonstrate linearity and wide spectral bandwidth operation.

1 Introduction

We have proposed recently a noise effective wavefront sensor for natural guide star called LIFT (Linearized Focal-plane Technique)[1]. The estimation by LIFT is based upon a single image taken at the focal plane of the telescope. The relation between the aberrations (decomposed on the Zernike polynomials[2]) and the intensity pattern is linearized to make computations easier and faster. Besides, we add a known phase offset ϕ_d in the wavefront sensor optical path to avoid undetermination [3]. The algorithm is detailed in the first part. We then show how to set the phase offset ϕ_d by means of simulation. Finally, we present the experimental results validating LIFT for the estimation of low-order modes : tip, tilt and focus.

2 LIFT : a phase-retrieval algorithm

Let ϕ be the aberrated phase and n the noise. The intensity pattern on the imaging sensor is :

$$I(\phi) = |\mathrm{FT}\{\underbrace{P\exp(i\phi_d)}_{P_d} \times \exp(i\phi)\}|^2 + n \tag{1}$$

ϕ can be decomposed on Zernike modes Z_i so that $\phi = \sum_i a_i Z_i$. Let $\mathbf{A}$ be the vector of coefficients a_i, and I be the function giving the intensity pattern in the focal plane. A first order Taylor expansion with respect to $\mathbf{A}$ yields:

$$\mathbf{I}(\mathbf{A}) - \mathbf{I}(\mathbf{0}) \simeq \sum_k a_k \mathbf{I}'_\mathbf{k}(\mathbf{0}) + \mathbf{n} \tag{2}$$

with $\mathbf{I}'_\mathbf{k}(\mathbf{0}) = \left.\frac{\partial \mathbf{I}(\mathbf{A})}{\partial a_k}\right|_{\mathbf{A}=\mathbf{0}}$ and $\mathbf{n}$ the noise vector.

With $\Delta\mathbf{I} = \mathbf{I}(\mathbf{A}) - \mathbf{I}(\mathbf{0})$, and $\mathbf{H}$ the matrix made of the vectors $\mathbf{I}'_\mathbf{k}$,we get:

$$\Delta\mathbf{I} = \mathbf{H}(\mathbf{0})\mathbf{A} + \mathbf{n} \tag{3}$$

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where $\mathbf{A}$ is to be estimated. We assume $\mathbf{n}$ is a zero mean gaussian noise, with $\mathbf{R_n}$ its covariance matrix. The maximum likelihood (ML) estimation of $\mathbf{A}$ is then given by:

$$\hat{\mathbf{A}}_{MV} = P_{MV}(0)\Delta\mathbf{I} \; ; \quad P_{MV}(0) = (\mathbf{H}(\mathbf{0})^t {\mathbf{R_n}}^{-1} \mathbf{H}(\mathbf{0}))^{-1} \mathbf{H}(\mathbf{0})^t {\mathbf{R_n}}^{-1} \tag{4}$$

The variance of the estimation error is given by :

$$var = Tr\{\langle \mathbf{E}\mathbf{E}^t \rangle\} = Tr\{(\mathbf{H}^t {\mathbf{R_n}}^{-1} \mathbf{H})^{-1}\} \tag{5}$$

with $\mathbf{E} = \mathbf{A} - \hat{\mathbf{A}}_{MV}$. The diagonal elements of $\langle \mathbf{E}\mathbf{E}^t \rangle$ are to the variance of the estimation error for each mode.

This solution is only valid for small aberrations. In order to extend the linearity domain, we use the following algorithm :

- Iteration 0 : $\boldsymbol{\Delta I} = \boldsymbol{I}(\boldsymbol{A}) - \boldsymbol{I}(\mathbf{0})$ et $\hat{\boldsymbol{A}}_{MV_0} = P_{MV}(0)\Delta\boldsymbol{I}$
- Iteration 1 : $\boldsymbol{\Delta I} = \boldsymbol{I}(\boldsymbol{A}) - \boldsymbol{I}(\hat{\boldsymbol{A}}_{MV_0})$ et $\hat{\boldsymbol{A}}_{MV_1} = P_{MV}(\hat{\boldsymbol{A}}_{MV_0})\Delta\boldsymbol{I} + \hat{\boldsymbol{A}}_{MV_0}$
- Iteration 2 : $\boldsymbol{\Delta I} = \boldsymbol{I}(\boldsymbol{A}) - \boldsymbol{I}(\hat{\boldsymbol{A}}_{MV_1})$ et $\hat{\boldsymbol{A}}_{MV_2} = P_{MV}(\hat{\boldsymbol{A}}_{MV_1})\Delta\boldsymbol{I} + \hat{\boldsymbol{A}}_{MV_1}$
- Iteration 3 : $\boldsymbol{\Delta I} = \boldsymbol{I}(\boldsymbol{A}) - \boldsymbol{I}(\hat{\boldsymbol{A}}_{MV_2})$ et $\hat{\boldsymbol{A}}_{MV_3} = P_{MV}(\hat{\boldsymbol{A}}_{MV_2})\Delta\boldsymbol{I} + \hat{\boldsymbol{A}}_{MV_2}$

The extention of the linearity domain with the number of iterations is shown on figure 1. Performing more than five iterations does not extend notably the linearity domain.

3 Astigmatism offset optimization with respect to noise propagation

The propagation of photon noise and read out noise on phase estimation error can be expressed as :

$$\sum_i \sigma^2(\hat{a}_i - a_i) = \overbrace{\Big(\sum_i \alpha_i\Big)}^{\alpha} \frac{1}{n_{ph}^{tot}} + \overbrace{\Big(\sum_i \beta_i\Big)}^{\beta} \Big(\frac{\sigma_e}{n_{ph}^{tot}}\Big)^2 \tag{6}$$

with n_{ph}^{tot} the total number of detected photons and σ_e the read out noise standard deviation (in ph.e^-). The coefficients α_i and β_i are respectively the photon noise and read out noise propagation coefficients for the Zernike polynomial Z_i.

The impact of the choice of the phase offset ϕ_d on noise propagation for tip, tilt and focus has been studied by computing the α and β coefficients for various values of ϕ_d according to eq. 5 (Fig. 2).

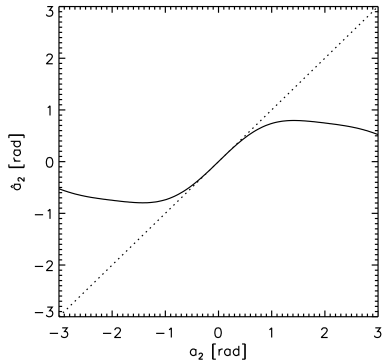
We see that the optimal value is around 0.5 radians. In what follows, we show the first experimental characterization of the linearity range of LIFT, in narrow band and wide-band.

4 Experimental validation

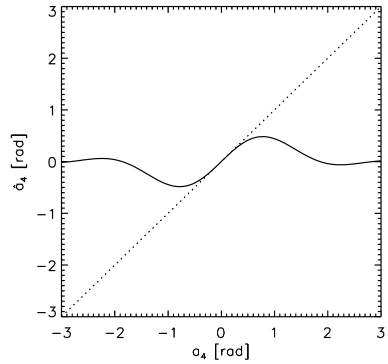
The experimental setup is shown in Fig. 3. The astigmatism offset is introduced by putting a tilted plate in front of an optical single mode fiber source. The tip/tilt and focus are inserted by translating the camera along X, Y or Z.

This offset and the focus to sense are monitored with a Shack-Hartmann wavefront sensor in the collimated space between the two lenses. The inserted tip is computed by a center of gravity. The linearity of LIFT on tip and focus was first validated with a laser diode at 635 nm (Fig. 4)

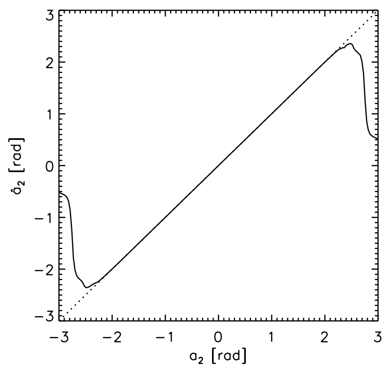
The estimation of focus was also achieved with a polychromatic source. Its FWHM is 200 nm for a central wavelength at 676 nm (Fig. 5). The inserted focus is given in radians at central wavelength..



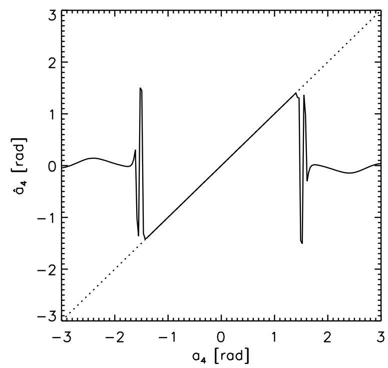
(a) Tip without iteration



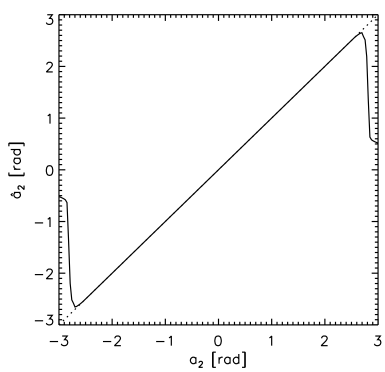
(b) Focus without iteration



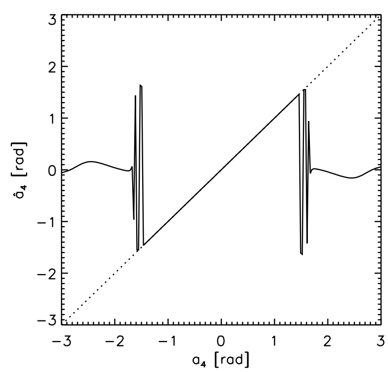
(c) Tip 3 iterations



(d) Focus 3 iterations



(e) Tip 5 iterations



(f) Focus 5 iterations

Fig. 1: Linearity domains with various numbers of iterations for tip and focus.

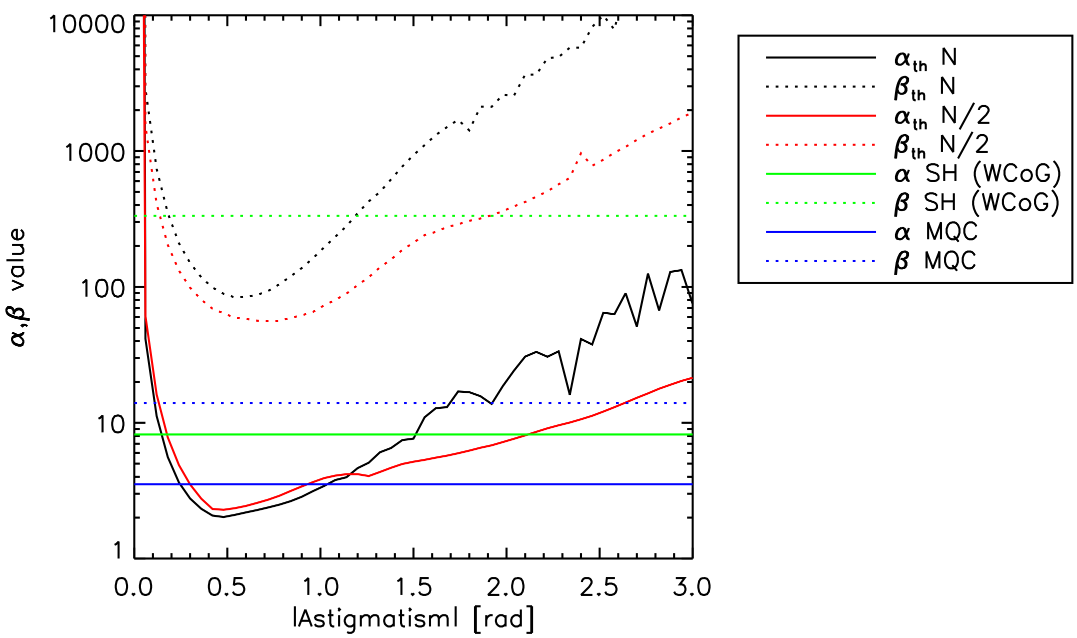


Fig. 2: α and β computed over 3 modes (tip+tilt+focus) for various astigmatism offset values ϕ_d and samplings (N = Nyquist frequency). Comparison with the Shack-Hartmann and the Modified Quad-Cell.

5 Conclusion

We have selected the optimum astigmatism offset by means of simulations. It appears that a 0.5 radians is optimum, and that a 10% margin is not critical. LIFT is therefore easy to implement on an existing system. We also present here an experimental proof-of-concept of LIFT, both on monochromatic and wideband sources. Current experimental developments are intended to confirm the analytical noise propagation we have presented here, which would complete the experimental validation of LIFT in lab. Next step would then be on-sky validation, hopefully in time for SPIE 2012!

References

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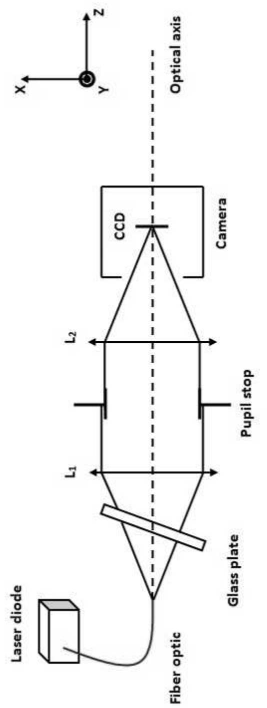

Fig. 3: Bench dedicated to the experimental validation of LIFT

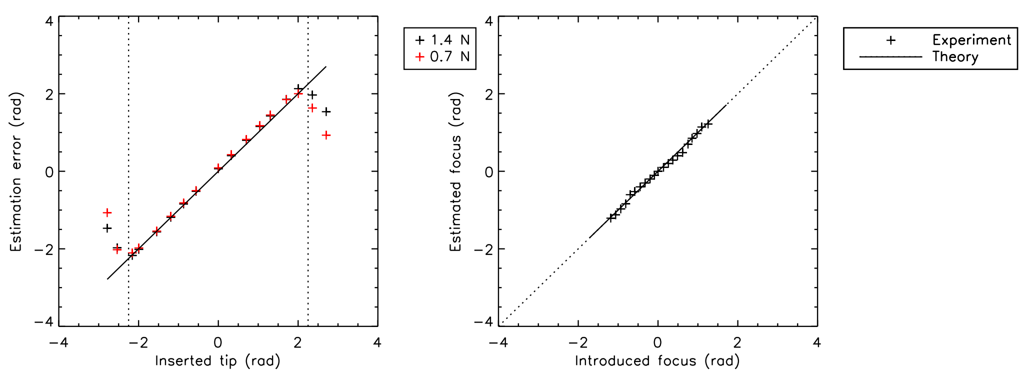

Fig. 4: Left : Estimation of a tip by LIFT with samplings at 1.4 and 0.7 times the Nyquist frequency (the dashed lines correspond to the expected linearity range). Right : Estimation of a focus by LIFT with a sampling at 0.9 times the Nyquist frequency.

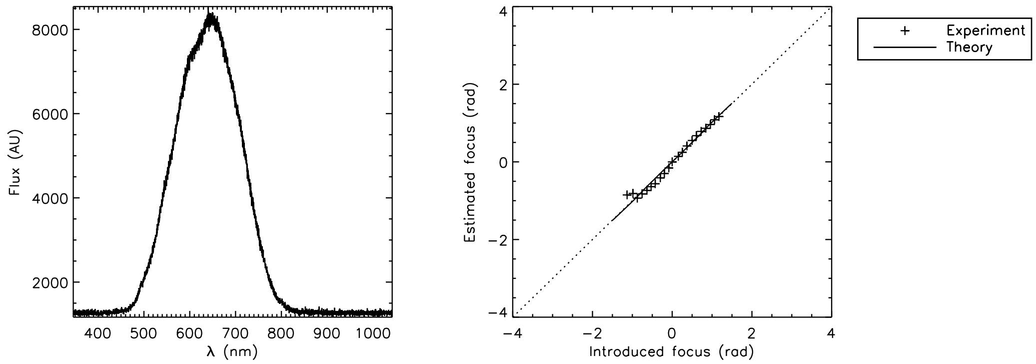

Fig. 5: Left : Spectrum of the wideband source. Right : Estimation of a focus by LIFT with a sampling at 0.9 times the Nyquist frequency (at the central wavelength), in large bandwidth.