

# Analytical vs. end-to-end numerical modeling of adaptive optics systems: comparison between the PAOLA code and the Software Package CAOS

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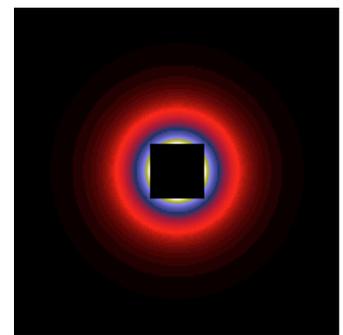
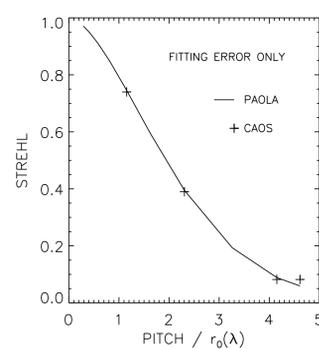
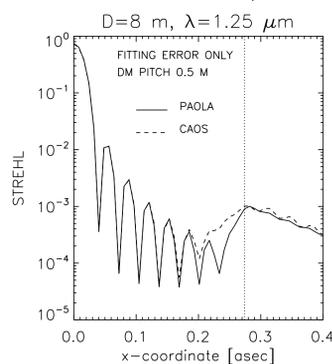
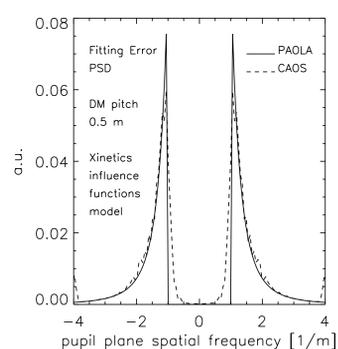
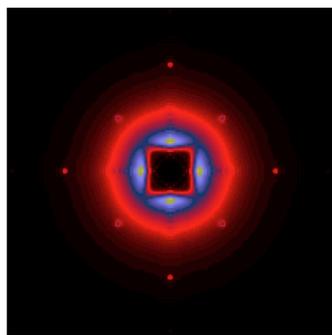
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**Abstract** We compare in this poster the analytical approach together with the so-called end-to-end approach in the framework of astronomical adaptive optics (AO) modeling. The two tools used for this purpose are well-known and already widely used within the astronomical AO community: PAOLA (Performance of Adaptive Optics for Large (or Little) Apertures, see Jolissaint, L., 2010, *J. Europ. Opt. Soc. Rap. Public.* 5, 10055) on the one hand, and the Software Package CAOS (see Carillet, M. et al 2005, *Mon. Not. R. Astron. Soc.* 356, 1263) on the other hand. This is indeed done in order to inter-validate the two codes, but also in order to search for trade-offs, or let's say optimal compromises, permitting then to face either exploratory researches or large instrumental project performance evaluations while combining as far as possible effectiveness and certainty. As preliminaries to the full comparison, we first test the fundamental fitting error and anisoplanatic error (equivalent in our test to the servo-lag error), and find a very satisfactory agreement. We then make a first attempt of full comparison by simulating within both models a complete 8m-class telescope AO system, varying the photon noise contributing to the whole wavefront sensing (WFS) noise.

## Preliminaries: Fitting Error

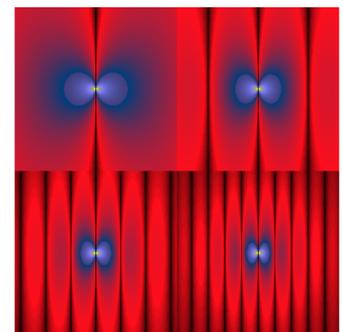
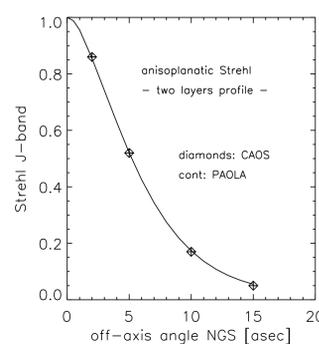
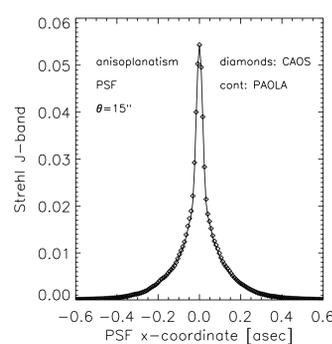
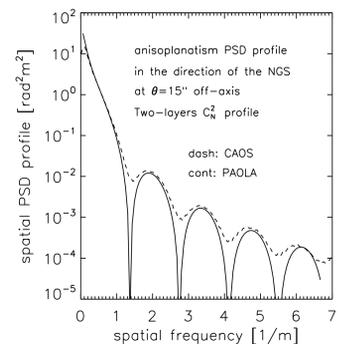
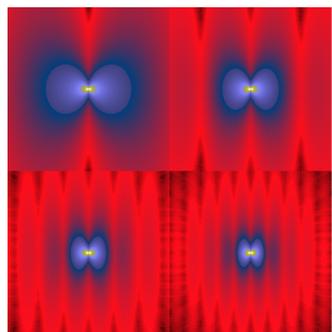
Within the CAOS-based model the residual wavefront is here simply the difference between the incoming turbulent atmosphere wavefront and its projection onto a deformable mirror (DM) influence function (IF) basis. A statistically-averaged point-spread function (PSF) is then deduced by running a large number of independent realizations of the turbulent atmosphere. Instead, the PAOLA model considers the Kolmogorov phase spatial power spectrum (PSD), set to zero inside the AO-corrected spatial frequency domain  $f < 1/(2 \text{ pitch})$  here for computing the fitting error (FE) structure function, from which the AO optical transfer function (OTF) is deduced, and so on up to the overall PSF. The turbulent atmosphere considered in both models is characterized by  $r_0=14.4$  cm and  $L_0=25$  m, with 1000 independent  $128 \times 128$  phase screens (with the addition of sub-harmonics) within the CAOS-based simulation. The DM pitches considered within the PAOLA model (0.5 m, 1 m, 1.8 m, and 2 m) correspond to sets of, respectively, 289, 81, 25, and again 25 IF within the CAOS model. This approach has its limits: (1) because the CAOS-based simulation assumes here a perfect WFS, aberrations above the AO cutoff frequency can be somewhat affected by the DM correction, while in a real system these high-order frequencies would remain as they are; (2) PAOLA assumes a perfect DM, totally correcting any phase within the AO cutoff frequency, which would need the IF to be *sinc*-like (the Fourier transform of a *sinc* being a door function), but of course IF are not *sinc* functions. These effects are clearly visible in the FE PSD shown below (left (CAOS) and right (PAOLA) 2D representations and first plot from the left): the CAOS PSD has features above the cutoff frequency, and shows a smooth transition to zero. Reversely, the PAOLA PSD shows a perfect Kolmogorov PSD above the cutoff frequency, and a perfect one-to-zero transition. Due to the structural relationship between the PSD and the PSF, these differences show up in the PSF (see second plot), but the overall Strehl (see last plot) is not really affected.



In order to make the PAOLA model closer to the CAOS result, we would need to implement into PAOLA a DM spatial transfer function model, a particularly interesting feature when the structure of the PSF within the cutoff frequency domain needs to be precisely known, as for instance when studying the performance of extreme AO with a possible coronagraph. Moreover, and in order to reproduce exactly the low-high spatial frequency transition, we would also need to better sample with respect to what is done by default: the sampling of the FE PSD appears to be too coarse and makes the PSF wings low-high frequency transition at a slightly different off-axis value than what is expected.

## Preliminaries: Anisoplanatic Error/Servo-lag Error

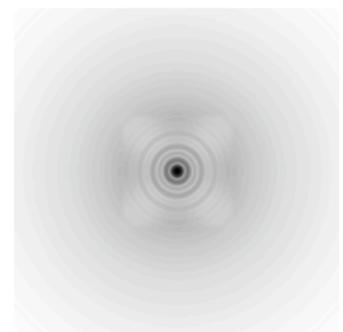
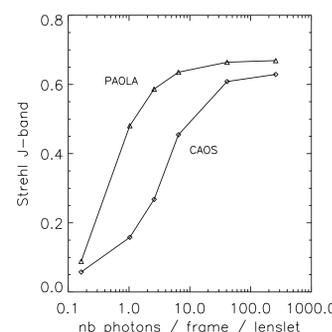
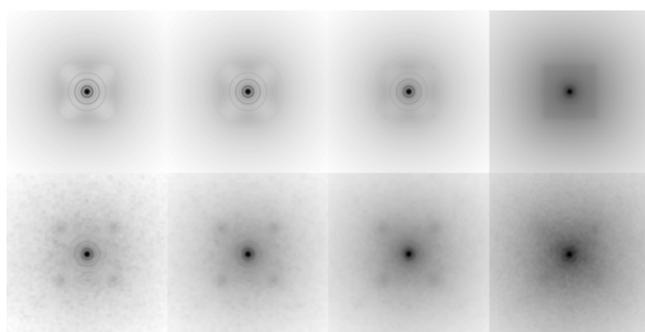
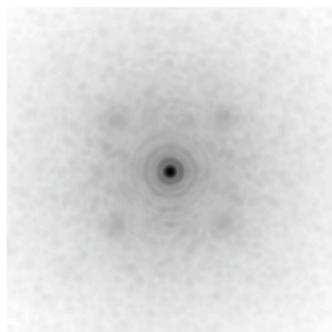
The second most frequent source of AO error is angular anisoplanatism and servo-lag error. These two errors are somewhat correlated: a lateral shift of the turbulent layers during one loop period (responsible for the servo-lag error) is equivalent to an angular shift of the phase when looking in two different directions. In the spatial frequency domain, the computation of the servo-lag error PSD uses the same principles than the computation of the anisoplanatic error (except that we also have the averaging of the phase during the WFS exposure) and as a consequence the structure of the servo-lag and anisoplanatic PSD looks the same. We test here the simplest mode, the angular anisoplanatism, for a 2-layers atmosphere (with 60% of the turbulence energy affected to an  $h=0$  km altitude layer and 40% to  $h=10$  km). Theory says that the anisoplanatic error modulation period is proportional to  $1/h\theta$ , where  $\theta$  is the off-axis angle, and this is well apparent in our two models: the left (CAOS) and right (PAOLA) 2D representations (for  $\theta=2''$ ,  $5''$ ,  $10''$ , and  $15''$ ) and first plot from the left show the PSD. The fact that the CAOS-based PSD does not drop as deep as the theoretical PSD is probably a sign of a lack of numerical convergence (1000 independent realizations only). The second plot shows the PSF profiles while the last plot shows the off-axis decrease of the Strehl. In all cases, the CAOS model and the PAOLA model agree.



It must be noted anyway that it is expected that for off-axis angles such that the on-axis and off-axis beams are totally separated, PAOLA should predict better Strehls than in reality, because since the analytical approach neglects the finite beam width (infinite aperture approximation), there always will be some (although low) level of correlation between the two beams. This is not apparent here because the off-axis angle are not large enough ( $165''$  would be necessary in this case).

## (First Attempt Of) Full Error Comparison, Featuring Wavefront Sensor Photon Noise

We are here entering the real exploration of expected differences between PAOLA and CAOS, each source of error being included (but angular anisoplanatism here). The PSF (right 2D representation and top row from PAOLA, left 2D representation and bottom row from CAOS with, for the rows and from left to right:  $m_V=14, 15, 16$ , and  $18$ ), and the predicted Strehl. Other relevant AO system parameters adopted are: wind velocities=8 m/s,  $16 \times 16$  sub-apertures Shack-Hartmann WFS (with  $8 \times 8$  px of angular size  $0''.128$  per sub-aperture, sensing at 620 nm with a bandwidth of 245 nm, neither read-out nor dark current noises considered), a 0.5-m-pitch DM (originally 289 IF for the CAOS model but filtered back to 206 modes after pseudo-inversion of the interaction matrix in order to eliminate modes which eigen-value is above a condition number of 10), and a global loop gain of 0.5.



These results are to be considered as they are: very preliminary. But several comments can already be expressed: (1) the impact of the waffle mode (the dots at the corners of the AO corrected domain) is missing in the PAOLA PSFs, which is not surprising as this effect is not modeled, yet; (2) the CAOS PSFs have not reached numerical convergence (1000 iterations of 1 ms but one only realization of the turbulent atmosphere — we are here rather simulating time evolution than statistical averaging as in the two previous cases), hence the speckle noise is still dominant (we would probably need at least 10 times more independent realizations in order to have a minimum of statistical averaging in addition to time evolution); (3) one of the main unknown is of geometrical nature: the equivalence between the spatial frequency cutoff, defined by the DM pitch, and the number of modes actually corrected within the CAOS model has to be clearly established — the PAOLA model does not have any modal filtering (everything is perfectly corrected up to the cutoff frequency), while within CAOS a selection of the DM modes to correct is mandatorily performed, based on the modes' eigen values; (4) moreover, both the number of modes to be corrected and the modal gain to be applied were absolutely not optimized within the CAOS model, while it should be done, in addition to the WFS time exposure actually, in order to have a better evaluation of the attainable Strehl. Considering the ensemble of remarks expressed here above should make the PAOLA evaluation less optimistic, and the CAOS one less pessimistic, leading a priori to a better tuning between the two models. We can however remark that when the noise error dominates, as it is the case with  $m_V=18$  (corresponding to 0.2 photons/frame/lenslet), we have some convergence of the two models.

Let us finally note that a by-product of this work is also the Software Package PAOLAC, an embedment of PAOLA within the CAOS problem-solving environment, which is being adapted to the last features of PAOLA (including close-loop)... CAOS problem-solving environment: <http://fizeau.unice.fr/caos> || PAOLA code: [laurent.jolissaint@aquilaoptics.com](mailto:laurent.jolissaint@aquilaoptics.com) || Any question: [marcel.carbillet@unice.fr](mailto:marcel.carbillet@unice.fr) or [laurent.jolissaint@aquilaoptics.com](mailto:laurent.jolissaint@aquilaoptics.com)