CuRe - A new wavefront reconstruction method for SH-WFS measurements

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1. Introduction

The AAO (Austrian Adaptive Optics) team is involved in developing wavefront reconstructors for SH- and Pyramid-WFS measurements utilizing the mathematical properties of the forward operators for these wavefront sensors. We focus mainly on direct reconstructors with complexity O(n) (where n denotes the number of active subapertures of the WFS) to make the reconstruction algorithms scalable for large telescopes. The Cumulative Reconstructor (CuRe) has been developed to reconstruct wavefronts ϕ from Shack-Hartmann sensor measurements s_x and s_y where

$$(s_x)_{ij} = \frac{\lambda}{2\pi|\Omega_{ij}|}\int_{\Omega_{ij}} \frac{\partial\phi(x,y)}{\partial x}dxdy, \qquad (s_y)_{ij} = \frac{\lambda}{2\pi|\Omega_{ij}|}\int_{\Omega_{ij}} \frac{\partial\phi(x,y)}{\partial y}dxdy.$$

An improvement of this algorithm, namely a domain decomposition method for enhancing reconstruction quality and improving the overall speed of the algorithm has been derived furthermore. A speed comparison with different wavefront reconstruction algorithms will point out the enormous gain of the new CuReD (Cumulative Reconstructor with Domain Decomposition) algorithm concerning numerical performance. An adaption of the CuReD to modulated Pyramid wavefront sensor measurements is possible, some results are presented in the talk *Numerical simulations of an Extreme AO system for an ELT* by Miska Le Louarn (ESO).

2. Basic Algorithm of the Cumulative Reconstructor

We recall the Cumulative Reconstructor as described in [7]: Consider a quadratic domain Ω where measurements $s_x[i,j]$, $s_y[i,j]$ of a Shack-Hartman wavefront sensor are given on subapertures $\Omega_{ij} \subset \Omega$. In the following we describe the reconstruction of the wavefront ϕ_x from the s_x measurements using the s_y measurements for the calculation of the trend only. The calculation of ϕ_y is similar.
Using the s_x measurements we reconstruct the chains in x-direction using

$$I_x\left[i_1+1, i_2-\frac{1}{2}\right] = I_x\left[i_1, i_2-\frac{1}{2}\right] + s_x\left[i_1+1, i_2\right],$$

for $i_1, i_2 = 1, \ldots, N$ starting with initial values $I_x\left[0, i_2-\frac{1}{2}\right] = 0$. We shift the chains such that the mean value is equal to 0 for each

$$I_{x0}\left[i_1, i_2-\frac{1}{2}\right] = I_x\left[i_1, i_2-\frac{1}{2}\right] - w_x\left[i_2-\frac{1}{2}\right],$$

where the w_x are the mean values of the lines

$$w_x\left[i_2-\frac{1}{2}\right] = \frac{1}{N}\left(\sum_{k=1}^{N-1} I_x\left[k, i_2-\frac{1}{2}\right] + \frac{1}{2}I_x\left[0, i_2-\frac{1}{2}\right] + \frac{1}{2}I_x\left[N, i_2-\frac{1}{2}\right]\right).$$

The trend lines are constructed using the mean of the slopes

$$t_y\left[i_2\right] = t_y\left[i_2-1\right] + \frac{1}{N}\sum_{i_1=1}^{N} s_y\left[i_1, i_2\right].$$

These will be shifted to mean value 0 and the values at the intermediate points corresponding to the crossings of the lines in the different directions are computed.

$$t_y\left[i_2+\frac{1}{2}\right] = \frac{1}{2}(t_y[i_2] + t_y[i_2+1]).$$

Finally, we shift the x-chains in such a way that the mean values match the y-trend line intermediate values and vice versa

$$\phi_x\left[i_1, i_2-\frac{1}{2}\right] = I_{x0}\left[i_1, i_2-\frac{1}{2}\right] + t_y\left[i_2-\frac{1}{2}\right].$$

Values at points $(x,y) \in \Omega$ are obtained via (rotated) linear interpolation, the wavefront at the corners of the subapertures, corresponding to the Fried geometry, can be computed as

$$\phi[i_1, i_2] = \frac{1}{4}\left(\phi_x\left[i_1, i_2-\frac{1}{2}\right] + \phi_x\left[i_1, i_2+\frac{1}{2}\right] + \phi_y\left[i_1-\frac{1}{2}, i_2\right] + \phi_y\left[i_1+\frac{1}{2}, i_2\right]\right).$$

The next figure shows the (independently) reconstructed chains and their alignment using the trendlines.

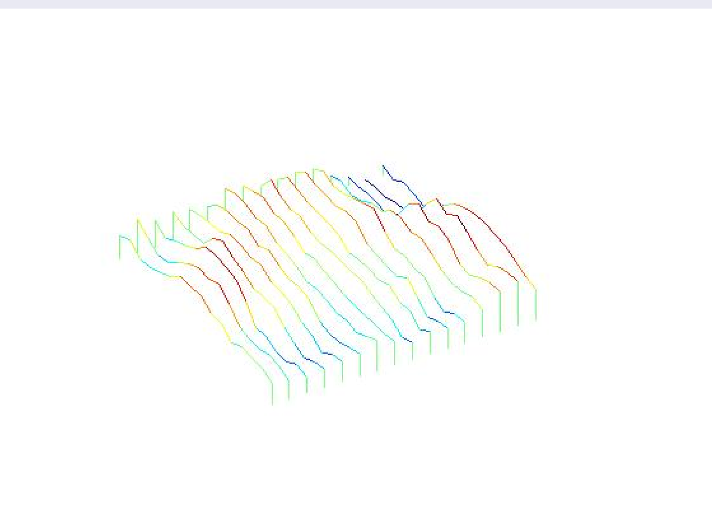

Figure: Chains I_{x0}

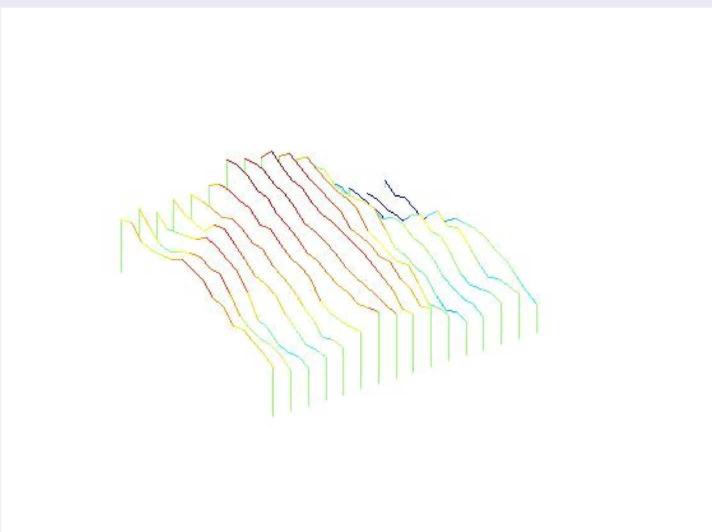

Figure: Shifted Chains ϕ_x

3. Adaption of the CuRe to circular apertures and central obstructions

Until now we have assumed a rectangular aperture where all measurements are given and valid. In real-life telescopes the aperture will be most likely annular, we have to cope with the existence of a central obstruction. An adaption of the CuRe to circular apertures with central obstruction is presented in [4] as well as a detailed analysis of the propagation of noise.

4. The CuRe with domain decompostition - CuReD

Although the CuRe is an extremely fast reconstructor with a high flexibility to adapt (i.e. spiders, mirror misalignment, dead actuators, different sensor geometries), it suffers from its bad error propagation properties if the number of sensor subapertures is large. In order to overcome this drawback a domain decomposition ansatz has been introduced. The telescope aperture is divided into rectangular subdomains and on each subdomain the CuRe is applied. The resulting reconstructions for each subdomain are coupled s.t. the overall wavefront reconstruction is smooth and its mean is zero. For details on the method see [5].
An additional benefit of the CuReD is its increased pipelineability (and parallelizability) compared to the CuRe.

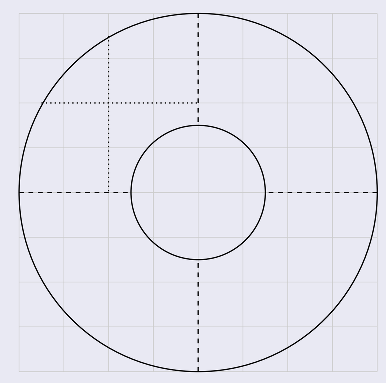

Figure: Decomposition of the domain Ω_T

5. Verification of reconstruction quality

The verification of the reconstruction quality has been performed within Octopus, an ESO AO end-to-end simulation environment. We have assumed a telescope diameter of D=42 meter, an 84x84 Shack-Hartmann sensor, the size of the central obstruction was 0.28D (11.76 meter) and the DM had 5402 active actuators. The Strehl has been evaluated for a wavelength of 2.2μm assuming a sampling frequency (time unit) of 1000Hz and a two frame delay for the DM control.

Comparison of MVM and CuReD for ESO standard atmosphere

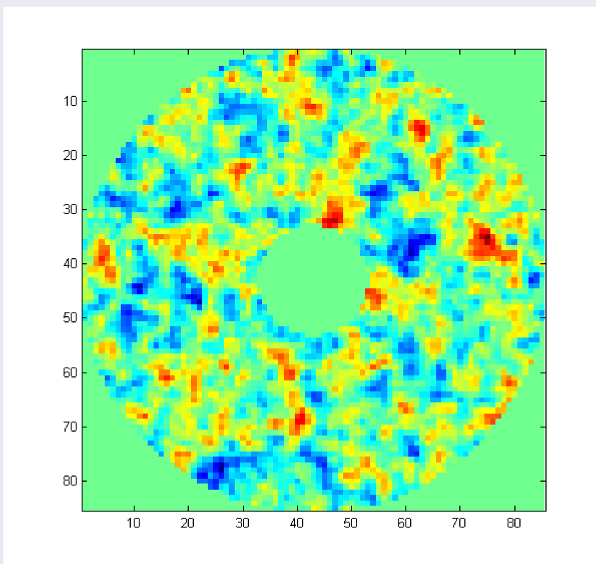

Figure: ϕ using CuReD

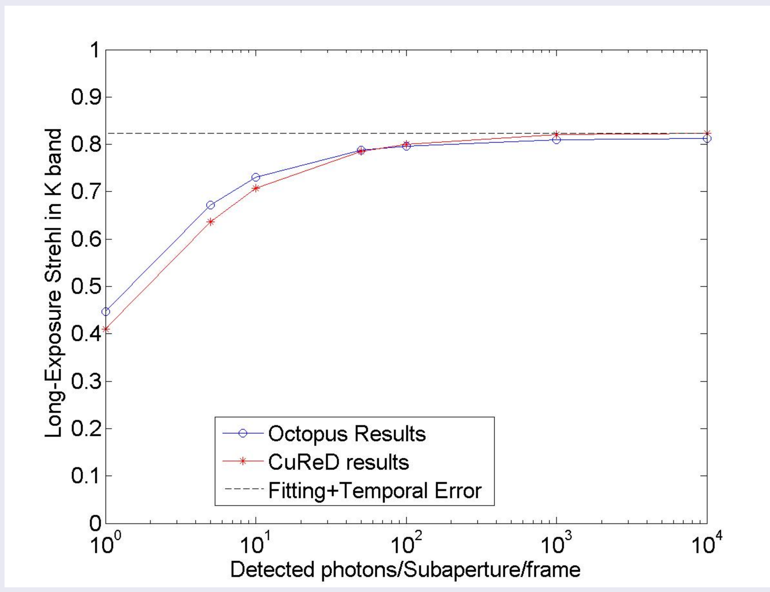

Figure: Varying photon flux level

6. RTC performance of the CuRe

The efficiency evaluation of the reconstruction methods has been performed on an ESO RTC-cluster by Lu Feng.

Algorithm	reconstruction time	CPU cores used
MVM	20 ms	8
FrIM	600 μs	1
sparse PCG	585 μs	48
FTR	190 μs	8
CuRe single core	130 μs	1
CuRe parallel	81 μs	2

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