Impact of the Cn^2 description on WFAO performance

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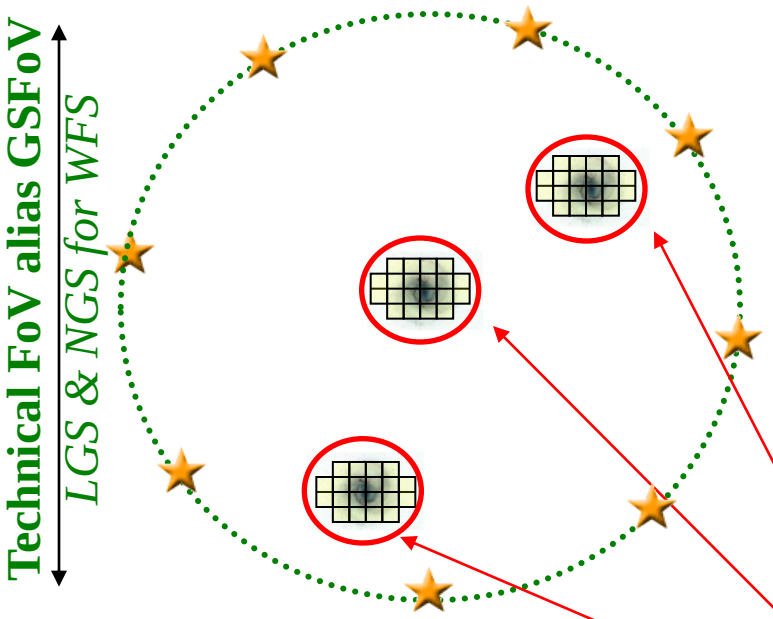
Outline

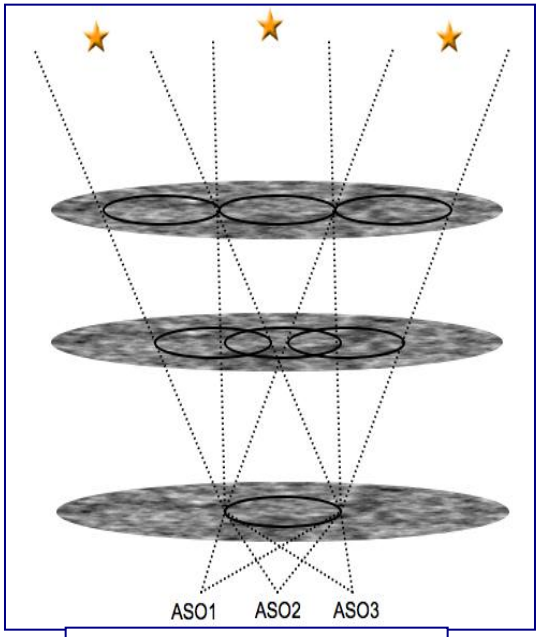
- Tomographic reconstruction in WFAO
- Impact of the input turbulent profile on tomographic reconstruction performance
- Impact of the error in the prior turbulent profile in the tomographic reconstructor
  - Error on the number of turbulent layers
  - Error on the altitude of the layers
  - Error on the Cn^2 profile

Tomographic reconstruction of the turbulence in WFAO



GSFoV = α_{GS}

Scientific FoV = SFoV



$$\tilde{\phi}_{\alpha}^{\mathbf{mes}} = \mathbf{M}\tilde{\phi}_{\alpha} + \mathbf{b}^{\mathbf{noise}}$$

$$\varphi_{corr}^{DM} = \underbrace{P_{concept}^{SFoV}}_{\text{DM projection Systemdependant}} \underbrace{W_{tomo}^{\alpha_{GS}} \cdot \Phi_{measurements}^{\alpha_{GS}}}_{\text{Tomgraphicreconstruction Common to all systems}}$$

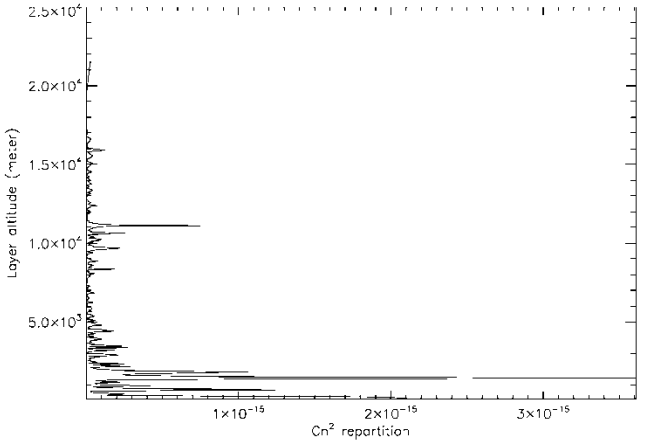
Turbulence related parameters

- Fried parameter r_0 : $= \left[0.42 \left(\frac{2\pi}{\lambda} \right)^2 \frac{1}{\cos\gamma} \int_0^\infty C_n^2(h) dh \right]^{-3/5}$

  Depends on the turbulent profile (C_n^2) and characterizes the strength of the turbulence

- Anisoplanatic angle $\theta_0 = 0.314 \frac{r_0}{\bar{h}}$ with $\bar{h} = \left(\frac{\int_0^\infty h^{5/3} C_n^2(h) dh}{\int_0^\infty C_n^2(h) dh} \right)^{3/5}$

- Turbulent profile defined by:
  - Given Cn^2 repartition
  - Given number of layers: N^l
  - Given altitude of the layers



Tomography related parameters

The true input turbulence
Initial profile with given number N^l of layers

→

The prior turbulent model
Model used in the reconstruction process with a given number of layers Nl

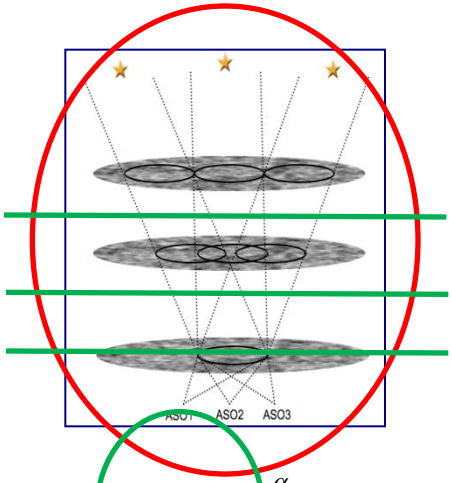
→

Tomographic error

- Impact of the Cn² profile:
  - As input for simulation of the turbulence
  - In the model in the reconstruction process

$$\sigma^2_{tomo,recons} = \sum_{i=0}^{n_{obj}} \left\| \Phi^{turb}(\beta_i) - \hat{\Phi}^{recons}(\beta_i) \right\|^2$$

- Need accuracy of Cn² knowledge
  - To limit the tomographic error
  - To perform good system design



$$\varphi^{DM}_{corr} = \underbrace{P^{SFoV}_{concept}}_{\text{DM projection}} \cdot \underbrace{W^{\alpha_{GS}}_{tomo} \cdot \Phi^{\alpha_{GS}}_{measurements}}_{\text{Tomgraphic reconstruction}}$$

Systemdependant
Common to all systems

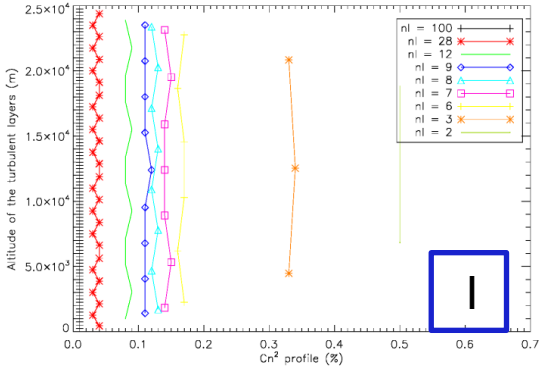
Tomographic error term

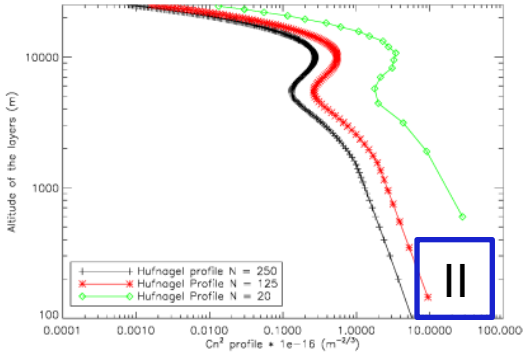
- $\sigma^2_{tomo,recons} = \sum ||\Phi^{turb}(\beta) - \Phi^{recons}(\beta)||^2$ where Φ = phase in the pupil
- $\Phi^{turb} = P^0(\beta^{obj}, N^0) \times \varphi^{turb}$
  - φ^{turb} = turbulent phase in the volume
  - β^{obj} direction of correction and N^0 = number of turbulent layers in the true input turbulence
- $\Phi^{recons}(\beta) = P^1(\beta^{obj}, N^1) \times W^{\alpha}_{tomo} \times [M \varphi^{turb} + n]$
  - N = number of turbulent layers in the model of turbulence
  - $W^{\alpha}_{tomo} = W^{\alpha}_{tomo}(P^1(\alpha^{GS}, N^1))$: tomographic reconstructor
  - $M = M(\alpha^{GS}, N^1)$: projector of the phase in the analysis direction
  - n = noise of the detector
- If $P^0 = P^1$
  - No model error on the turbulent profile
  - Number of turbulent layers in the profile perfectly known
- If $P^0 \neq P^1$
  - Model error: unknown turbulent profile: N^1, Cn^2 repartition and layers position

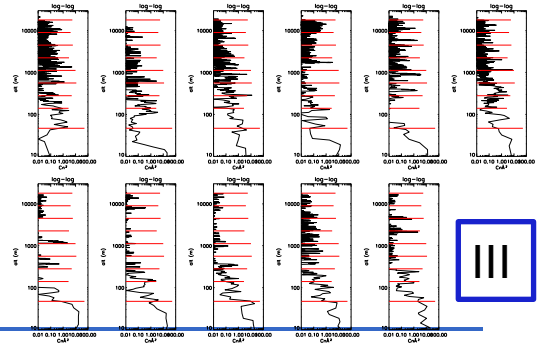
→ Two types of error to study

Impact of the true input turbulent profile

- Study of the tomographic reconstruction for a 42 m telescope
  - LTAO with MMSE reconstruction
  - 1 DM conjugated with the pupil: 84 x 84 actuators
  - 1 on axis correction
  - Simulation thanks to a Fourier based code
- System characteristics
  - Number of GS: 6 or 3
  - GS FoV : 1' or 2'
- Turbulence characteristics
  - R_0 = 0.57 m @ 1.6 µm, Θ_0 = 2'
  - N^0 can vary between 1 and 250 layers between 0 and 25 km
  - Three turbulent profiles
    - Cn^2 constant: type I
    - Hufnagel type: type II
    - Profile obtained from balloon measurements: type III



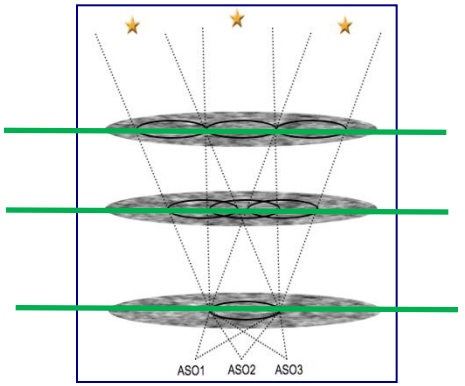




Different turbulent models BUT same turbulent conditions (r_0 ,Θ_0) and same system characteristics

Impact of the number of layers in the true input turbulent profile

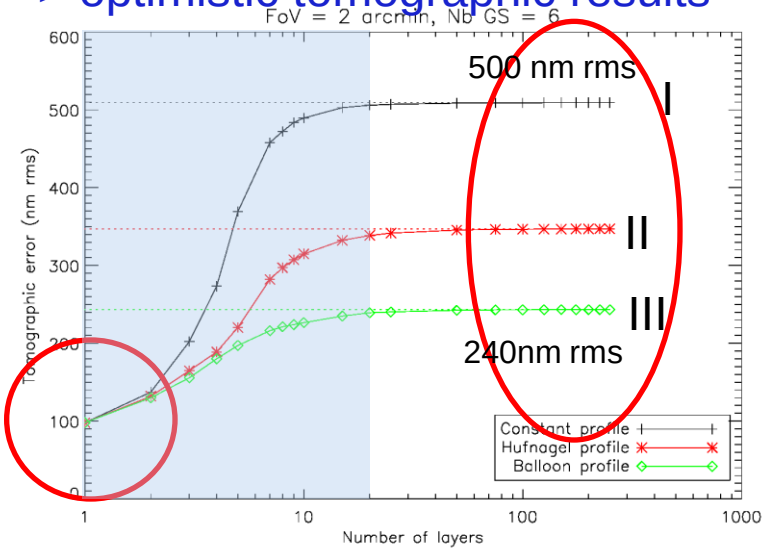
- Impact on the tomographic error $\sigma^2_{tomo,recons}$ of the number of layers N^0 in the turbulent profile
- $P^0 = P^1$: Turbulent profile perfectly known, $N^0 = N^1$ turbulent layers



- Increase of $\sigma^2_{tomo,recons}$ with N^0
- Not dependent of the turbulent profile
- Same first point because same turbulent conditions

→ The performance depends on the structure of the profile for same r_0 and Θ_0

Under-estimation of $\sigma^2_{tomo,recons}$
=> optimistic tomographic results

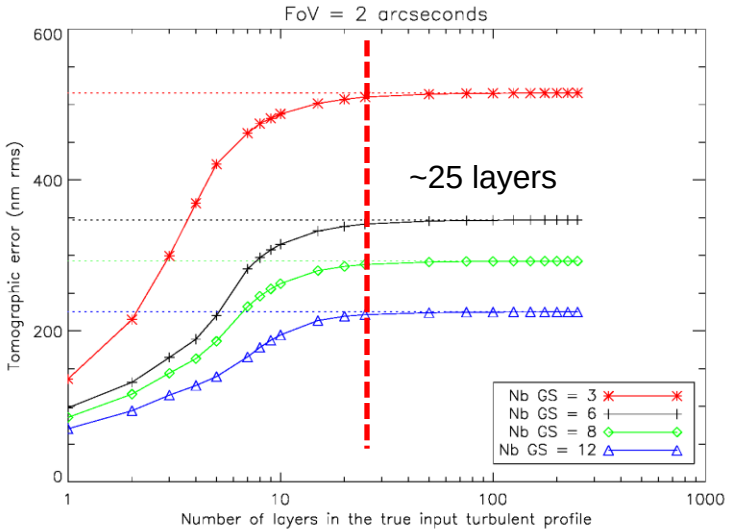


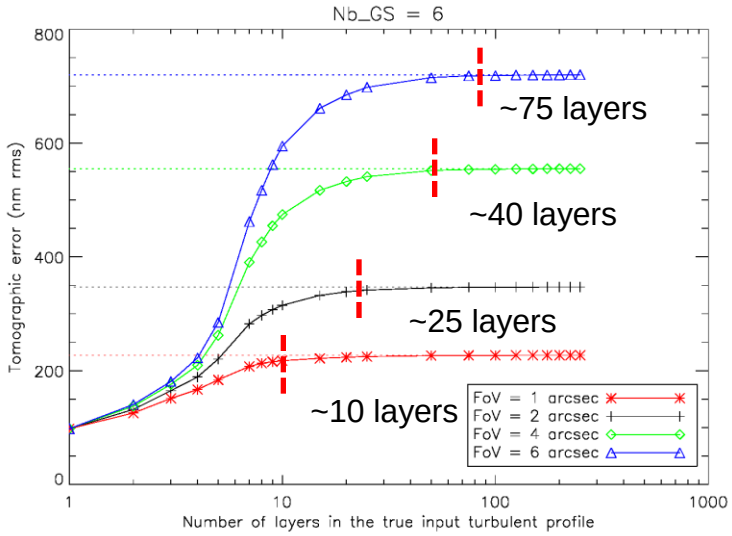
Impact of the system parameters

- $P^0 = P^1$: Impact of the system parameters (FoV, number of GS)
- Turbulent profile: constant profile type I

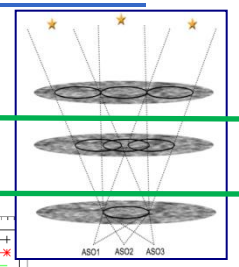
- $\sigma^2_{tomo,recons}$ increases with the GS FoV
- $\sigma^2_{tomo,recons}$ decreases with the number of GS
- The beginning of the plateau depends on the FoV

The larger the FoV is, the more you need turbulent layers in your input turbulence



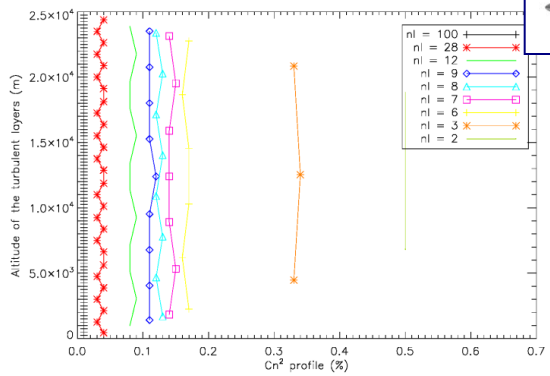


Impact of the number of layers in the prior turbulent model

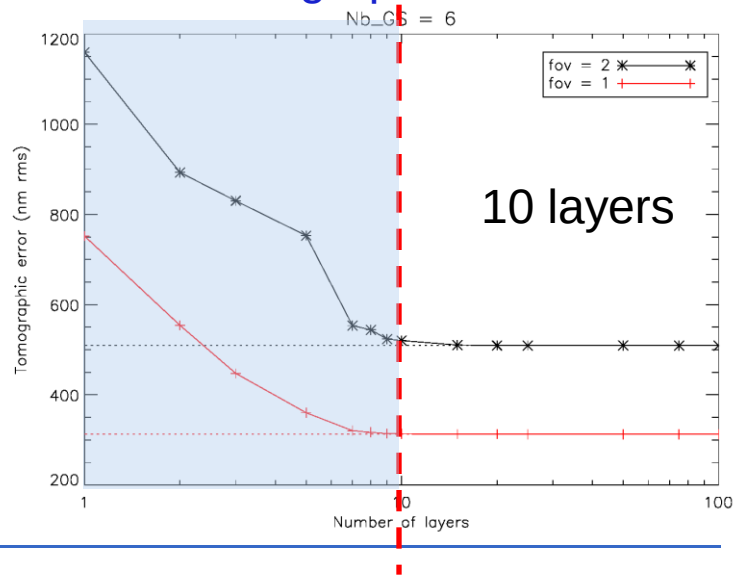


- $P^0 \neq P^1$ and $N^0 \neq N^1$
- Impact of the number N^1 of layers on $\sigma^2_{tomo,recons}$
- Input turbulent profile P^0 : type I N^0=100 layers
- Same r_0 ,Θ_0, number of GS and FoV
- Prior model P^1 : number of layers between N^1 = 2 and 100 layers

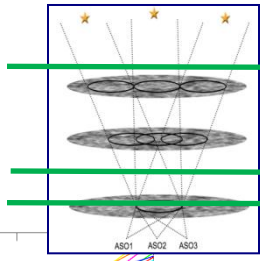
→ Need at least 10 layers in the reconstructor to avoid over-estimation of tomographic error



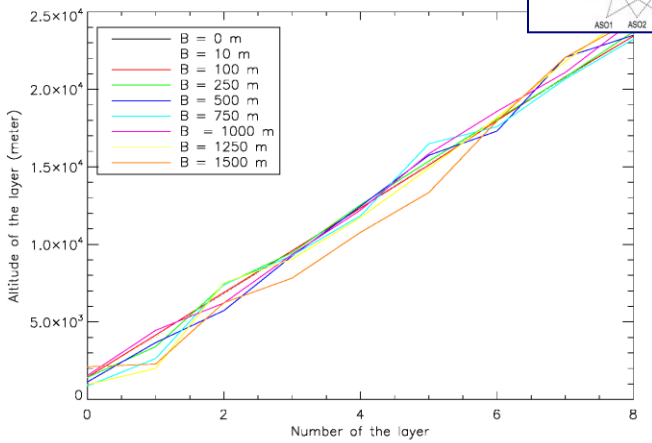
Over-estimation of $\sigma^2_{tomo,recons}$ => pessimistic tomographic results



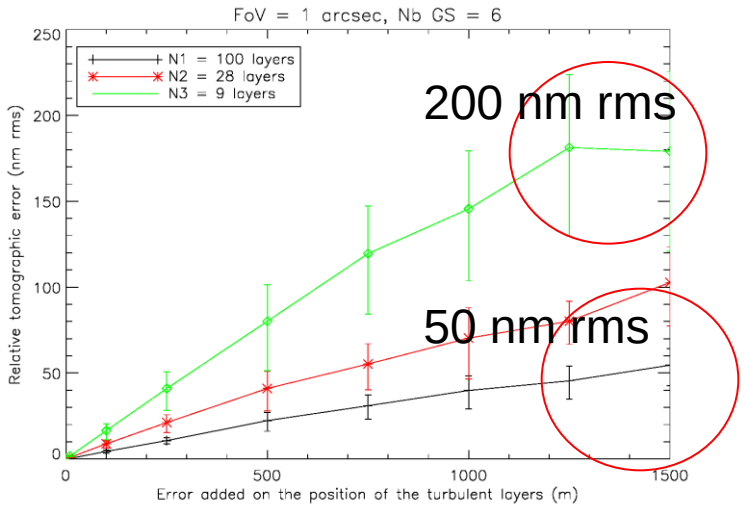
Impact of an error on the altitude of the layers in the prior turbulent model



- $P^0 \neq P^1$ and $N^0 = N^1$
- Three cases studied: $N^0 = N^1 = 9$, 28 or 100 layers
- Error of the altitude of the layer:
  - $h^0 \neq h^1$
  - $h^1 = h^0 + B > 0$ where B is a random value between 0 and1500 m



- $\sigma^2_{tomo,recons}$ increases with the error on the altitude
- $\sigma^2_{tomo,recons}$ increases when the number of layers decreases



→ less sensitive to the error when the model contains lots of turbulent layers

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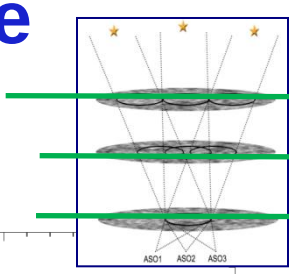
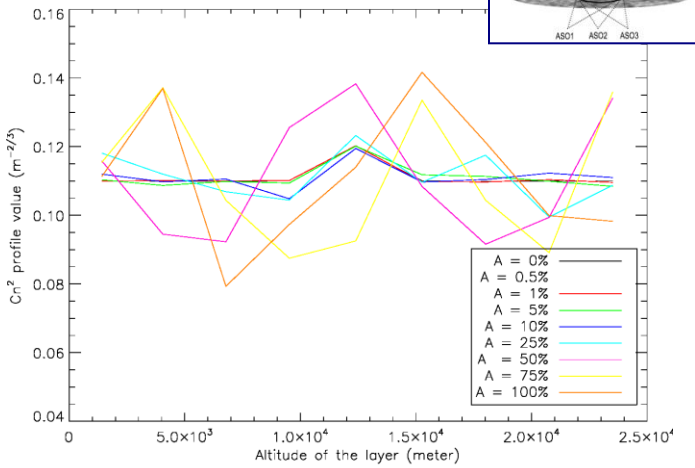
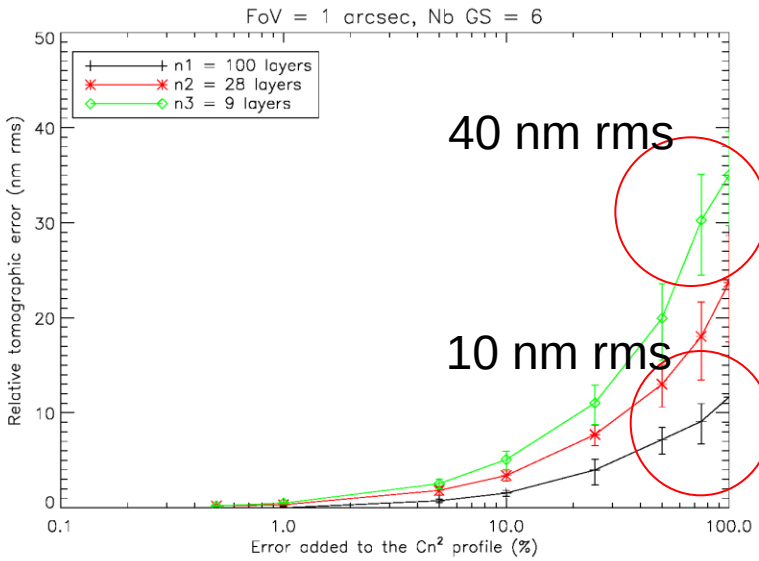
Impact of an error of the r_0 per layer in the prior turbulent model



- $P^0 \neq P^1$ and $N^0 = N^1$
- Three cases studied: $N^0 = N^1$ = 9, 28 or 100 layers
- Error of the Cn^2 repartition:
  - $Cn^{2,0} \neq Cn^{2,1}$
  - $$C_n^{2,1} = \frac{C_n^{2,0} + A}{\int C_n^{2,0} + A} > 0$$

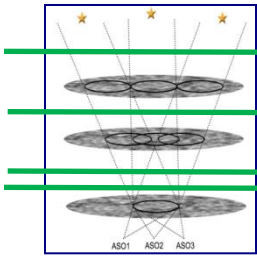
  where A is a random value of a percent error on the Cn^2 value
- $\sigma^2_{tomo,recons}$ increases with the error on the Cn^2
- $\sigma^2_{tomo,recons}$ increases when the number of layers decreases





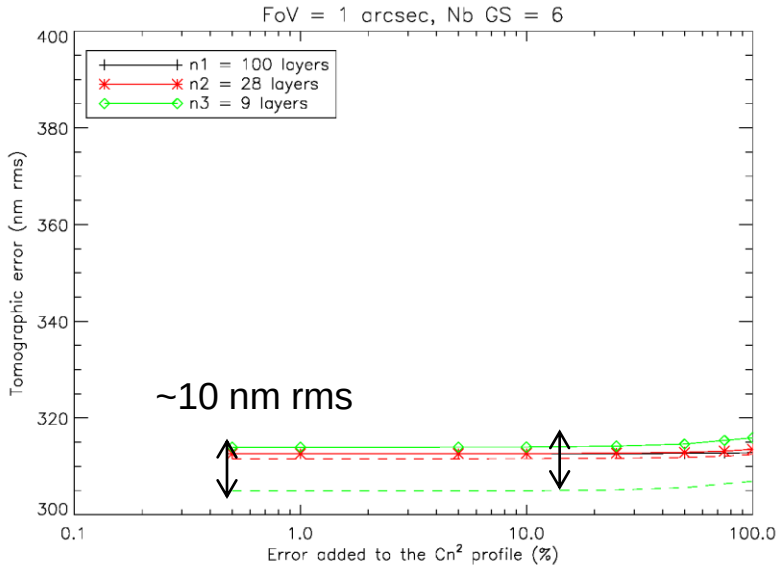
→ less sensitive to the error on r_0 per layer, especially if the profile contains lots of turbulent layers

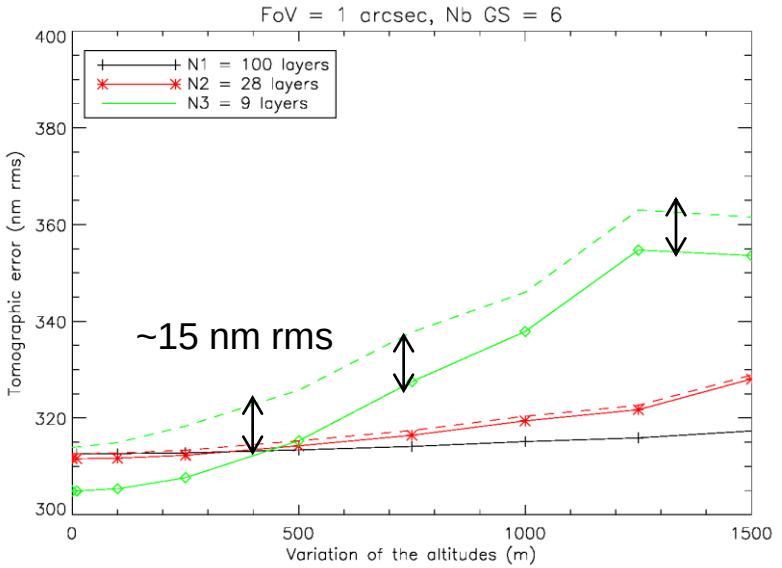
Impact of several model error in the prior turbulent model



- $P^0 \neq P^1$ and $N^0 \neq N^1$
- N^0 = 100 layers for the true input turbulence
- N^1 = 100, 28 or 9 layers for the prior turbulent profile
- Error on the altitude and on the Cn^2 repartition: $h^0 \neq h^1$ and $Cn^{2,0} \neq Cn^{2,1}$
- Same behaviors:
  - Less impact of error on Cn^2 profile
  - Error increases when N decreases

→ Errors are decorrelated





Conclusion on part 1

- Impact of the input turbulent profile for ELT tomographic AO system:
  - Performance simulation and system design
    - Need to take into account more complex turbulent profiles to avoid under-estimation of the tomographic error
    - The number of turbulent layers varies with the size of the GS FoV
  - For Site testing and choice
    - Need to take into account more parameters than r_0 ,Θ_0
    - Need high resolution Cn^2 profile data
    - Development of new dedicated systems for site testing ?

Conclusion on part 2

- Impact of model errors in the tomographic reconstructor
  - Need at least 10 layers in the reconstructor
  - Error on the position of the layers : important if few number of layers in the profile
  - Error on the r_0 per layer: not important
  - Increasing the number of layers in your reconstructor => reduce sensibility to absolute knowledge of layers position
  - Model errors (number of layers, position and repartition) are decorrelated => should simplify your error budget analysis and minimization

- Need to have a trade-off between the RTC complexity (number of layers in the reconstructor) and the required accuracy on Cn^2 prior knowledge