Refined Adaptive Optics simulation with wide field of view for the ELT

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Abstract. Refined simulation tools for wide field AO systems (such as MOAO, MCOA or LTAO) on ELTS present new challenges. Increasing the number of degrees of freedom (scales as the square of the telescope diameter) makes the standard codes useless due to the huge number of operations to be performed at each step of the AO loop process. This computational burden requires new approaches in the computation of the DM voltages from WFS data. The classical matrix inversion and the matrix vector multiplication have to be replaced by a cleverer iterative resolution of the Least Square or Minimum Mean Square Error criterion (based on sparse matrices approaches).
Moreover, for this new generation of AO systems, concepts themselves will become more complex: data fusion coming from multiple Laser and natural guide stars will have to be optimized, mirrors covering all the field of view associated to dedicated mirrors inside the scientific instrument itself will have to be coupled with split or integrated tomography schemes, differential pupil or/and field rotations will have to be considered, etc
All these new entries should be carefully simulated, analyzed and quantified in terms of performance before any implementation in AO systems.
In this paper we present a new E2E simulator, developed to deal with all these specific ELT challenges. It is based on an iterative resolution of the linear model with high number of degrees of freedom (using the sparse matrix) and includes new concepts of filtering and coupling between LGS and NGS to effectively manage modes such as tip / tilt and defocus in the entire process of tomographic reconstruction.

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1 Introduction

Its evident that the potential application of Adaptive Optics, both for imaging and for laser energy transfer, suffer from the lack of suitable reference source. In an Adaptive Optics System we care about the data beacon measured by a wave front sensor in order to compensate for the effect of turbulence. This measured path must be in the isoplanatic patch [1] as the observed object, so that the system performance can still be nearly equal to the diffraction limited-value.
Its clear that Laser Guide Stars will be required in any desired direction in order to limit for the anisoplanatism effect.

The first studies of methods that doesn't rely on a natural sources for measuring the atmospheric distortion, was invented at Itek by R. A. Hutchin. Starting from the 'ray method' in 1978, where the Laser pulses are fired from multiple subapertures of the telescope, to the idea of creating an artificial source in the atmosphere in 1981 [2]. In the same time, the same ideas was published by Foy and Labeyrie using the concept of pulsed Laser backscatter from a region of the atmosphere.
Unfortunately their are additional error sources that must be taken into account once using a Laser beacon instead of natural reference source:

1. The cone effect since the source is formed within the earth's atmosphere and so the beam do not sample the full telescope beam or any turbulence above the Laser source.
2. The fluctuation of the sodium layer's makes us lose information about the exact LGS altitude. Or Tip Tilt and deFocus are affected by the position of the LGS in the sodium level. For Tilt compensation a fixed source is required.
3. Their will be backscatter into the telescope pupil if the laser wavelength matches the resonance frequency of the atmosphere sodium layer, a $0.5\mu m$ must match the line D_2 of the mesospheric sodium layer.

In this paper we focus on the two first mentioned problems, the cone effect and the TTF (Tip Tilt and deFocus) indetermination. In fact, the LGS emits a spherical wavefront distortion, which is used as an estimate of a plane wavefront distortion. So the target is corrected using the spherical wave. This difference called focus anisoplanatism is characterized by d_0 representing an aperture diameter-sized quantity that measures the magnitude of the effect of focus anisoplanatism, its given by [3]:

$$d_0 = \lambda^6 cos^3(\psi)[\int dhC_n^2(h)F(h/H)]^{-3/5}$$

$F(h/H)$ is a combination of hypergeometric functions.
In section 1 we present and discuss a new technique of filtering Tip/Tilt and defocus, we demonstrate that the HO could have the same answer on the WFS as the TT and se discuss the influence of basis changing on the LO. In section 2 we present an optimal solution for coupling between LGS and NGS in order to manage Tip and Tilt modes in the entire process of tomography reconstruction. Section 3 contains simulation result done in the frame of the EAGLE project. The entire studies and simulations are done using an E2E simulator developed to deal with all challenges coming with new telescope's generations , it's based on an iterative resolution of the linear model using a sparse routines.

2 Tip, Tilt and deFocus indetermination

Tip Tilt determination is a fundamental problem for adaptive optics system using a Laser Guide Star. A laser beacon projected from the ground can not be used as a reference to measure the position of a scientific object. This is due to the random motion of the LGS because of the atmospheric turbulence, in another word, the tilt in the outgoing path differ from the tilt on the incoming path. Moreover, the fluctuation of the sodium layer will affect the knowledge of where the LGS is focalized. For all those reasons, it is necessary to remove the tip/tilt and defocus to the spherical wave front measurements S.
$S = D_{WFS}\phi$ is the linear model, D_{WFS} is the answer of the wave front sensor to the incident spherical wave front.

The objective is to find the vector slopes S_{OTTF} that excludes the TTF components. So its convenient to write S_{OTTF} as

$$S_{OTTF} = \text{S - } \alpha S_{TIP} - \beta S_{TILT} - \gamma S_F$$

Where αS_{TIP},βS_{TILT} and γS_F are the measurements to the contribution of TTF in the spherical incident wave-front.
The estimate of the wave front given the WFS measurements is an inverse problem, must be resolved using an iterative method with the proper conditioner, and to avoid the complexity on inverting the ill-conditioned ASO matrix we find the voltage that minimize the residual slopes :

$argmin||M_{in}u - S||^2$, Where $M_{in}$is the interaction matrix.
And the restores phase is given by : $\hat{\varphi} = Fu$, F are the influence functions.

The vector slopes can be divided to an orthogonal space and parallel one to the modes in question.

$$S = S_{//} + S_{\perp}$$

The measurements out of TTF is the projection of S to $S_{\perp}$ which is given by:
$S_{\perp} = D_{\perp}(\varphi_{\perp} + \alpha Z_2 + \beta Z_3 + \gamma Z_4) = D_{\perp}\varphi_{\perp} = D_{\perp}\varphi$
$D_{\perp} = (ID - MM^{\dagger})D_{ASO}$ Where M is a transformation matrix to an orthogonal space which excludes TTF. Let us discuss all the possible way to find the matrix M.

1. The first way for filtering TTF is to set the mean slopes to zero $\bar{S} = 0$, then $MM^{\dagger}$ can be defined as the matrix that calculate the mean of the slopes vector, so that we can write $S_{OTTF} = (ID - \frac{1}{n}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & 1 & \vdots \\ 1 & \cdots & 1 \end{bmatrix})S$

Unfortunately, setting the slopes mean to zero will not remove the overall tip/tilt components. In fact, this procedure will move the modes TT but it also affects the other modes having contribution to the mean slopes. so we are going to recreate TT from the higher order, (see Figure1)

2. As it is shown above the method of the mean slopes will not be the optimal solution for an estimated phase excluded from TTF. Or and because of the reconstructor described above, we lost the bijection between slopes and phase so the answer of the WFS could be the same for different modes. For all those reason M should contain the answer of the WFS to all the modes

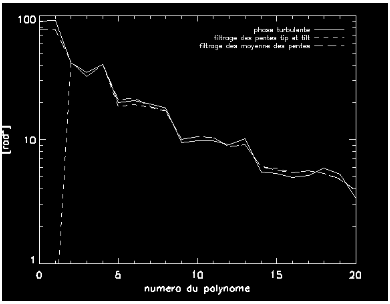

Fig. 1. Projection of the turbulent phase onto the Zernike basis(solid line), the filtered phase (long dashed line) and a phase without TTF(dashed line)

in question TTF in our case, and the three lines of $M^\dagger$ are excluded from $M^\dagger_{system}$which contain the answer of the WFS to all the modes generated by the system :
$M^\dagger_{system} = [S_{TIP} S_{TILT} S_F \cdots S_{nmode}]^\dagger$
S_{nmode} is the slope of the n mode created by the system, so we propose to take advantages of the KL basis (see Figure2 left), that transform the phase into finite orthogonal functions. $MM^\dagger$ is then given by:

$$MM^\dagger = [S_{TIP} S_{TILT} S_F][S_{TIP} S_{TILT} S_F]^\dagger$$

Or if we consider that D_{WFS} is condition and so invertible we show in (Figure2 right) that we don't lost bijection between phase and slopes, so $M^\dagger$ is the result of the generalized inverse of M, no need to calculate the biggest matrix $M^\dagger_{system}$.

3 Fusion Data LGS/NGS

In this section we present an optimal way for fusion the measurements from NGS and LGS. We are going to discuss two different way for interaction of LGS/NGS data in an AO system: Integrated Tomography gonna be discuss in this section (see Figure3), and Split Tomography , unless this method separate the measurements in too loops so we lost information's on NGS/LGS correlation wavefront (see Figure4).

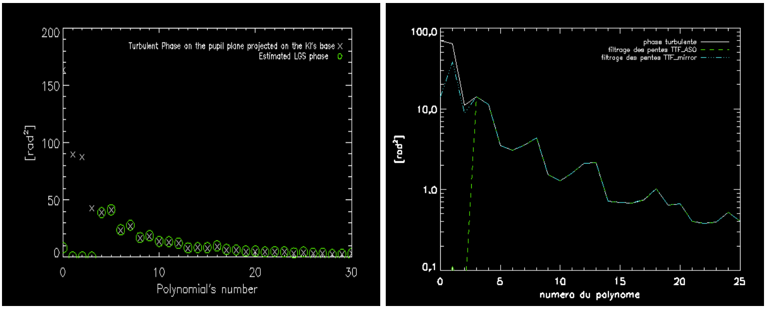

Fig. 2. Left :Projection of the filtered phase onto the KL basis. Right:Comparison between a reconstructor used in our simulation(blue line) and the reconstructor via the WFS(green line), in the case where $M^{\dagger}$ is the is the generalized inverse of M

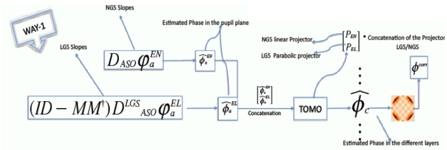

Fig. 3. Integrated Tomography

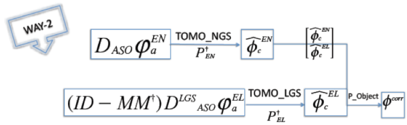

Fig. 4. Split Tomography

We focus our study on the Integrated Tomography as an optimal solution in terms of signal management.

– LGS projected pupil phase: Working on an open loop simulation without considering the temporal aspect, we randomly generate a turbulent phase with a Kolmogorov DSP. For an optimized volume reconstruction given the LGS measurements, we build a bilinear interpolation matrix M_{expand} formed by n layers block that expend the generated turbulent phase in the ratio $\frac{Dim}{PUP_{LGS}}$ where Dim is the telescope diameter represented in pixel, PUP_{LGS} is the LGS pupil.

$$\begin{matrix} L_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & L_n \end{matrix}$$

This expended phase is then projected to the pupil plane using a hyperbola rectangular sparse projector P_{EL} in order to project all the contribution of the phase at the different layers for all the ASO directions (see Figure5).

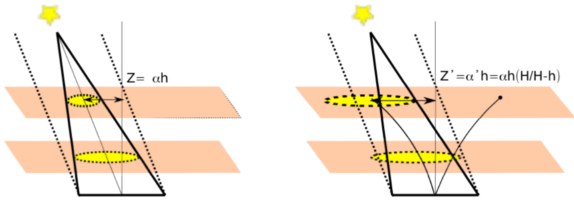

Fig. 5. LGS hyperbolic projector

The center of the LGS footprint at a given altitude moves away at the factor of $\frac{1}{h}$

$$Z(h) = \alpha h \frac{H}{H-h}$$
$$Z(h) = -\alpha h - \frac{\alpha H^2}{h-H}$$
$$Y(h) = \frac{K}{X}$$

Where K = $-\alpha H^2$, X = h-H, $Y = Z + \alpha H$

$$\Phi^{\alpha}_{pup} = P_{EL} M_{expand} \phi^{L^{tur}_n}$$

$$\begin{pmatrix} \vdots \\ \Phi^{\alpha}_{pup} \\ \vdots \end{pmatrix} = \begin{pmatrix} P^{L_1}_1 & \cdots & P^1_{L_n} \\ & \ddots & \\ \vdots & & \vdots \\ P^{\alpha}_l & \cdots & P^{\alpha}_l \end{pmatrix} \begin{pmatrix} L_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & L_n \end{pmatrix} \begin{pmatrix} \vdots \\ \phi^{L_n}_{tur} \\ \vdots \end{pmatrix}$$

- NGS projected pupil phase: The turbulent generated phase is projected in the pupil plan using a linear sparse projector $P_E N$, so the the pupil NG phase is the the contribution from all the layers n as given by : $\Phi^{\alpha}_{pupNG} = P_{EL}\phi^{N_n^{tur}}$
- Estimated Phase in the layers: Given the measurements LGS/NGS, we can estimate $\hat{\Phi}^{LGS}_{OTTF}$ and $\hat{\Phi}^{NGS}$ both phases are then concatenated. Based on a iterative resolution of the linear model, the estimated phase at the different layers is given by: $\hat{\Phi}^{L} = R\hat{\Phi}^{\alpha}$

$$R = (P^T P + \sigma^2 \gamma C_{\varphi}^{-1})^{-1} P^T$$

P is the concatenation of the LGS/NGS projector

For the Kolmogorov turbulence spectrum, the covariance matrix and so its inverse will be none-sparse and of full rank so to calculate C_{φ} we adopt the approximation proposed by [4]: $\varphi^{-1} \approx \gamma\nabla^4$, where γ is a constant, ∇^4 is the bi-laplacian operator.

4 E2E Simulator

4.1 High degree's of Freedom

Working with sparse Operations with Yorick/IDL, originates from a collection of IDL/C routines [5], we develop an E2E Simulator based on an iterative resolution of the linear model with high number (see Figure4.1) of degrees of freedom (using the sparse matrix) (see Figure7) , includes new concepts of filtering and coupling between LGS and NGS to effectively manage modes such as tip / tilt and defocus in the entire process of tomographic reconstruction, (See section 3).

5 Conclusion and Perspective

In this paper we present an E2E simulator code based on the sparse routine and developed in order to manage the high degrees of freedom

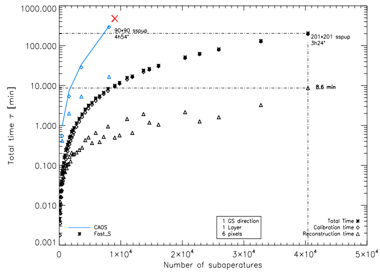

Fig. 6. Efficiency of E2E-s comparing to CAOS, where we show the calibration time in terms of sub-apertures.

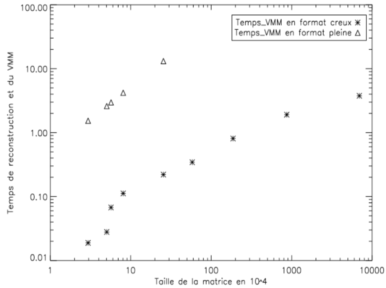

Fig. 7. Gain in memory storage and so the calculus time while using sparse matrices.

entered withe the E2E generation. We discuss an optimal solution for filtering TTF while using the spherical wave front as source of measurements. Materials for fusion data LGS/NGS is also presented in section 3, and how to merge and concatenate the measurements and the LGS/NGS projector. We are currently working on the application of this tool in the frame of the EAGLE project, article PHD dissertation.

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